RESOURCES MATERIAL FOR TEACHERS

CLASS X – MATHEMATICS

Workshop on

‘Developing resource material for teaching of Mathematics for classes IX-XII’

Venue: Kendriya Vidyalaya Sangathan, ZIET Mysore

(20th April to 25th April 2015)
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FOREWORD

There is an adage about Mathematics: “Mathematics is the Queen of all Sciences”. This adage exemplifies the significance, scope and importance of mathematics in the realm of sciences. Being a ‘Queen’, as a subject, Mathematics deserves to be adored and admired by all. But unfortunately, this subject is perceived by the students as a most difficult subject. Not only in India, but across the globe, learning of the subject creates trepidation.

The perception about this subject being difficult in India is rather surprising as ours is a land of great mathematicians like Ramanujan, Bhaskara, Aryabatta et al. The origin and accomplishments of these great men should be a source of inspiration to both students and teachers alike. Yet, as the truth being otherwise, making concerted efforts to identify the reason for perceived fears, initiate suitable damage control and undertake remedial measures assume paramount importance. Kendriya Vidyalaya Sangathan, as a pace setting educational organization in the field of School Education which always strives to give best education to its students, thought it fit to take a pioneering step to empower teachers through teacher support materials. In-service education too strives to do the same. Yet, providing Teacher Resource Material in a compact format with word, audio and video inputs is indeed a novel one.

In the name of teacher resources the internet is abound with a lot of materials: books, audio and video presentations. Yet their validity and usability being debatable, a homemade product by in-house experts could be a solution. Hence, in response to KVS (HQ)’s letter dated 03.03.2015 on the subject “Developing Resource Material for Teaching of English and Maths”, a six-day workshop was organized at ZIET, Mysore from 20-25 April 2015. The task allotted to ZIET, Mysore by the KVS is to prepare the resource materials for teachers of mathematics teaching classes IX to XII.

In the workshop, under the able coordinator ship of Mrs. V. Meenakshi, Assistant Commissioner, KVS, Ernakulam Region, four material production teams were constituted for preparing materials for classes IX, X, XI and XII separately. Mr. E.Ananathan, Principal, KV, No.1, Tambaram of Chennai Region headed Class XII Material Production team; Mr.E.Krishnamurthy, Principal, KV,NFC Nagar, Gatakeshar of Hyderabad region headed Class XI Material Production Team; Mr. Siby Sebastian, Principal, KV, Bijapur of Bangalore Region headed Class X Material Production Team and Mr.Govindu Maddipatla, Principal, KV, Ramavermapuram, Trissur of Ernakulam Region headed Class IX Material Production Team. Each team was aided ably by a group of five teachers of Mathematics. After a thorough discussion among KVS faculty members and Mrs.Sharada, TGT (Maths), an invited faculty from Demonstration Multipurpose School, RIE, Mysore on the ‘Reference Material Framework’ on the first day, the teams broke up to complete their allotted work. Their tireless efforts which stretched beyond the prescribed office hours on all the six days helped complete the task of preparing four Teacher Resource Booklets – one each for classes IX, X, XI and XII in a time-bound manner.

Even a cursory glance of the index shall reveal the opt areas of support that the Resource Booklet strives to provide to the teachers of mathematics. The entire material production team deserves appreciation for the commendable work they did in a short period of six days. It is the earnest hope of KVS that the effective use of the Resource Materials will serve the purpose of real teacher empowerment which will result in better classroom teaching, enhanced student learning and above all creating in the minds of the students an abiding love for the subject of mathematics.

- Dr. E. Thirunavukkarasu  
Deputy Commissioner & Course Director
PREFACE

“When you establish a destination by defining what you want, then take physical action by making choices that move you towards that destination, the possibility for success is limitless and arrival at the destination is inevitable.” — Steve Maraboli


The Sponsored Four Trained Graduate Teachers and one Post Graduate Teacher in Mathematics from Bangalore Region were allotted two/ three topics from syllabus of Class X to prepare Resource Material for teachers under the heads:

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As per the given templates and instructions, each member elaborately prepared the Resource Material under Fourteen heads and presented it for review and suggestions and accordingly the package of resource materials for teachers were closely reviewed, modified and strengthened to give the qualitative final shape.

The participants shared their rich and potential inputs in the forms of varied experiences, skills and techniques in dealing with different concepts and content areas and contributed greatly to the collaborative learning and capacity building for teaching Mathematics with quality result in focus.

I would like to place on record my sincere appreciation to the Team Coordinator Mr. Siby Sebastian Principal K.V.Bijapur, the participants Mrs. B. Banumathy PGT (Maths) KV NAL Bangalore, Mr. W. S. Balasubramanian TGT(Maths) KV RWF Yelahanka, Mrs. Kausalya Bai TGT(Maths) KV CRPF Yelahanka, Mrs. Sarada VJ TGT (Maths) KV Mysore, Smt. Priya G Nath TGT(Maths) KV No.1 Jalahalli, the Course Coordinator Mr. Arumugam PGT (Phy) ZIET Mysore and the members of faculty for their wholehearted participation and contribution to this programme.

I express my sincere thanks to Dr. E. T. Arasu, Deputy Commissioner and Director KVS, ZIET, Mysore for giving me an opportunity to be a part of this programme and contribute at my best to the noble cause of strengthening Mathematics Education in particular and the School Education as a whole in general.

My best wishes to all Post Graduate Teachers and Trained Graduate Teachers in Mathematics for very focused classroom transactions using this Resource Material to bring in quality and quantity results in the Class X Examinations.

Mrs. V Meenakshi

Assistant Commissioner, Ernakulum Region
Guidelines to Teachers

The Resource Material has been designed to make learning Mathematics a delightful experience catering to every kind of learner. As the learners are introduced to a fascinating variety of tools, and participate in meaningful, fun filled activities their Mathematics competence will grow exponentially. Activities that cater to different learning styles such as problem solving, reasoning and proof, analytical, logical etc. are thoughtfully placed in the Resource Material.

1. Expected Learning Outcomes:
In this section, the expected learning outcomes are enlisted chapter-wise and these are expected to be realized among the students on completion of particular chapter. The teachers have to design their teaching programme which includes mathematical activities, variety of tools and other mathematical tasks. Teachers may prepare their Power point presentations and use in their regular teaching in order to realise the desired outcomes.

2. Concept mapping in VUE portal:
The concept mapping works under Visual Understanding Environment portal, which can be downloaded freely from “Google”. A concept map is a type of graphic organizer used to help teachers/students organize and represent knowledge of a subject. Concept Maps begin with a main idea (or concept) and then branch out to show that the main idea can be broken down into specific topics. The main idea and branches are usually enclosed in circles or boxes of any Geometrical figure, and relationship between concepts indicated by a connecting line linking new concepts. Each concept is embedded into the box, and those concepts in the form of power point presentation, word document, videos web links etc are uploaded in the same folder.

How to use a concept Mapping?
The teacher can use as a Teaching Aid for explaining the holistic view of the topic. It can be used as revision tool. Concept maps are a way to develop logical thinking and study skills by revealing connections and helping students see how individual ideas form a larger whole. These were developed to enhance meaningful learning in Mathematics. It enhances metacognition (learning to learn, and thinking about knowledge). It helps in assessing learner understanding of learning objectives, concepts and the relationship among those concepts.

Download VUE portal from google and click on this icon to view the content embedded.

3. Three levels of graded exercises including non-routine questions:
In this section, selected questions collected from various reference books and are arranged in graded manner, in order the child attempt and learn mathematics in that order. Questions are given in three levels of nature easy, average and difficult respectively. These exercises facilitate the teacher to assign home/ practice works to the students as per their capabilities.

4. Value based questions:
In this section, Value based questions are given in each chapter with an objective to make a student aware of the moral values along with the value of problem solving. It is an endeavour to inculcate value system among the students and make them aware of social, moral values and cultural heritage of our great nation. It is expected that the students develop the values like friendliness, Honesty, Initiative, Compassion, Loyalty, Patience, Responsibility, Stability, Tactfulness and Tolerance along with problem solving skills and other applications.

5. Error Analysis and remediation:
It has been observed that the students commit a few common errors. In order to overcome this issue, teachers have listed, chapterwise, all possible common errors likely to be committed by the students and suitable measures to overcome those errors.
5. **Question Bank:**
   The questions were prepared chapter wise and kept in order for guiding the students suitably in their process of learning. Two sets of sample papers were also included for better understanding of the pattern of the Board Question Paper including weightage of marks.

6. **Activities/ Projects and Practicals:**(Developed by CDAC Mumbai and Amritha University under grant from Ministry of IT Govt. of India)

   The Online Labs is based on the idea that Maths lab experiments can be taught using the Internet, more efficiently and less expensively. The labs can also be made available to students with no access to physical labs or where equipment is not available owing to being scarce or costly. This helps them compete with students in better equipped schools and bridges the digital divide and geographical distances. The experiments can be accessed anytime and anywhere, overcoming the constraints on time felt when having access to the physical lab for only a short period of time.

   The features include;

   Content aligned to the NCERT/CBSE curriculum.

   - Mathematics Labs for Class 9 and 10.
   - Interactive simulations, animations and lab videos.
   - The concepts and understanding of the experiment.
   - The ability to perform, record and learn experiments - anywhere, anytime, and individualised practice in all areas of experimentation.

   **Robocompass** is a Euclidean Geometry Software on a Virtual 3D Paper. With just a handful of commands the users can explore basic constructs to understand proofs involving congruence, similarity, ratio, reflection, etc. Robocompass is fully integrated with Google Drive, so teachers can save their demonstrations and give construction assignments as homework to their students all by sharing their files through Google Drive. In this section, the solutions to the exercises under Geometric Construction in NCERT Textbooks of class IX is designed using the Robocompass software.

7. **Power point presentation and Video clips:**

   As educators, our aim is to get students get energized and engaged in the hands-on learning process and video is clearly an instructional medium that is compelling and generates a much greater amount of interest and enjoyment than the more traditional printed material. Using sight and sound, video is the perfect medium for students who are auditory or visual learners. Video stimulates and engages students creating interest and maintaining that interest for longer periods of time, and it provides an innovative and effective means for educators to address and deliver the required curriculum content.

   PowerPoint is regarded as the most useful, accessible way to create and present visual aids. It is easy to create colourful, attractive designs using the standard templates and themes; easy to modify compared to other visual aids, such as charts, and easy to drag and drop slides to re-order presentation. It is easy to present and maintain eye contact with a large audience by simply advancing the slides with a keystroke, eliminating the need for handouts to follow the message.

   The Resource material contains Power Point Presentations of all lessons of Class X and Video clips/links to Videos of concepts for clarity in understanding.

   Please double click on it to view the Power Point Presentation.
8. **Reference Web links:**

What is EDMODO?

- Free, privacy, secure, social learning platform for teachers, students, parents, and schools.
- Provides teachers and students with a secure and easy way to post classroom materials, share links and videos, and access homework and school notices.
- Teachers and students can store and share all forms of digital content – blogs, links, pictures, video, documents, presentations, Assign and explain online, Attach and links, media, files, Organize content in Edmodo permanent Library, Create polls and quizzes, Grade online with rubric, Threaded discussions- prepare for online learning!

9. **KVS JUNIOR MATH OLYMPIAD:**

KVS conducts JMO for KV students on first Sunday of September every year. A Student studying in class X who secured A2 or higher grade in Mathematics of class IX examination/A student studying in class XI with Mathematics as an elective subject (irrespective of stream opted) who secured A2 or higher grade in Mathematics in class X examination can appear/A student studying in class XII with Mathematics as an elective subject who secured 80% and above marks in Class XI in Mathematics can also appear. There will be a common question paper for all the participants.

**Syllabus:** The syllabus for Mathematical Olympiad (regional, national, international) is pre-degree college Mathematics. The areas covered are arithmetic of integers, geometry, quadratic equations and expressions, trigonometry, co-ordinate geometry, system of linear equations, permutations and combinations, factorization of polynomials, inequalities, elementary combinatorics, probability theory and number theory, finite series and complex numbers and elementary graph theory. The syllabus does not include calculus and statistics. The major areas from which problems are given are number theory, geometry, algebra and combinatorics. The syllabus is in a sense spread over Class XI to Class XII levels, but the problems under each topic involve high level of difficulty and sophistication. The difficulty level increases from JMO →RMO → INMO→IMO.

In this section, JMO sample papers are given with hints to solve them.

10. **Sample papers:**

In this section, blue print and sample papers are included for SA1 and SA2 which helps teachers to give practice tests in the board pattern.

11. **Formative Assessment:**

It is a process used by teachers and students as part of instruction that provides feedback to adjust ongoing teaching and learning to improve students’ achievement of core content. As assessment for learning, formative assessment practices provide students with clear learning targets, examples and models of strong and weak work, regular descriptive feedback, and the ability to self-assess, track learning, as well as to set a goal. Formative assessments are most effective when they are done frequently and the information is used to effect immediate adjustments in the day-to-day operations of the course. Assessment is not formative unless something is “formed” as a result of interpreting evidence elicited. It informs teacher where the need/problem lies to focus on the problem area. It helps teachers to give specific feedback, provide relevant support and plan the next step. It helps student
identify the problem areas, provides feedback and support. It helps to improve performance and provides opportunity to improve performance. Peer learning can be encouraged at all stages with variety of tools.

**Online Assessments:** Online Assessments are effective 21st century tools which empower the teachers to extend the class room beyond the four walls. In this technological era administering online assessments are very easy and immediate feedback is obtained. Free web portals such as Edmodo, Hot potato, Education Weekly etc. help teachers. This book includes an insight into these web portals. Online assessment is used primarily to measure cognitive abilities, demonstrating what has been learned after a particular educational event has occurred, such as the end of an instructional unit or chapter. Formative assessment is used to provide feedback during the learning process. In online assessment situations, objective questions are posed, and feedback is provided to the student either during or immediately after the assessment. http://www.halfbakedsoftware.com/hot_pot.php

12. **Tips and Techniques to score better:**
   This book includes tips and techniques for the students to score better. These tips will certainly help the teachers to guide their students for better achievements.

13. **Tips and Techniques in Teaching Learning process:**
   The Tips and Techniques included in this book for better Teaching learning Process will certainly be handy for the teachers who use this book.

14. **Annexure folder:**
   This folder is a collection of all soft copies which are embedded in the section ‘Concept Mapping in VUE portal’.

   ‘Power Point Presentations’ of the lessons are included for classroom teaching.

   Further Video clippings of a few problems and concepts are included.

**Feedback:**

The Post Graduate Teachers and Trained Graduate Teachers are requested to use this material in Classroom transaction and send feedback to Mrs. V. Meenakshi, Assistant Commissioner, Ernakulam Region.

minakviswa@gmail.com

BEST WISHES!
TEACHING OF MATHEMATICS

Introduction:
India is the land of Aryabhatta, Ramanujam and the like – great luminaries in the field of mathematics. Yet, this is one subject that our students dread the most. “It is a nightmarish experience learning this subject, and even the very thought of the subject sends jitters” is a common refrain of school-going students. Not only in India, a developing country, but also in other countries, be those come under the category of underdeveloped or developed, Mathematics is a subject of fear among our students. Both parents and students feel that class room teaching of the subject alone is not adequate for learning it effectively.

The Parent’s worries:
The parents are worried lot. Getting a ‘good’ Mathematics teacher, whatever it means, is big problem. The ones they get in the ‘market’ are not of any big help to their children; yet they are left with no option but to depend upon either school teachers or coaches from outside. Class room teaching is woefully inadequate in enabling the children acquire confidence and interest in the subject. The quantum of individual attention paid to solving the problems of students in this subjects being almost nil, making them get through the examination is a challenge. “Something needs to be done to arrest the rut being set in Mathematics teaching” is the common prayer of parents.

Why do children consider mathematics a difficult subject?
When you ask the teachers of Mathematics as to why Mathematics is considered as a difficult subject, the answers you get are neither logical nor scientific. Here are a few samples:
The subject requires more of students study time than other subjects (why?)
Students fail to practice problems (what is the reason?)
The subject requires long hours of work, involving practice and drill (why is it so?)
This subject is different from other subjects (in what ways?) Though these answers might partially tell us the reason why the subject is detested by many, they fail to throw any light on the psychological prerequisites, if any, specially required for learning this subject.

What goes on in Mathematics classes?
A peep into Mathematics classes and a bit of observation of the ways in which Mathematics is taught by Mathematics teachers reveal a pattern which is as follows:

Introduction of the new topic: the teacher speaking in general terms for a few minutes about the topic on hand if it happens to be the beginning of the topic.

Working out problems on the black board either by the teacher himself or by calling out a student to do it: If the teacher does the problem on the board, one can see him doing it silently or lip-reading the steps involved in it. If the student does the problem on the board, either continuous interruption or silent observation of the teacher can be seen to be taking place in the class.
Occasional fielding of questions by the teacher on the problem area:

When students raise doubts on the steps written how the steps have been arrived at etc., the teacher either clarifies the doubts or tells them to go through the steps again.

Leaving a large number of problems for the students to solve: Often after having solved one or two problems given under the exercise questions on the black board (at times those worked out problems happen to be given as model problems in the text book), teachers tend to leave a large number of remaining problems as home work to students.

What is wrong with the Existing Teaching Practices?

A critical analysis of what is wrong with the existing practices of Mathematics teaching is of prime importance. The analysis of commonly existing Mathematics teaching practices is given below:

Introduction of the Topic

Introduction of any new topic is done in not more than 5-10 minutes duration. This duration is not enough. You cannot throw light on the conceptual framework of a topic integrating the related concepts learned in classes down below in a span of 5 to 10 minutes. The teacher cannot say much in such a short duration. What actually transpires in Mathematics classes in the name of introducing topic can be illustrated with an example: In the illustration, I have taken here on the topic ‘quadratic equation’ taught by a teacher which goes like this. “We are going to learn quadratic equations today. Any equation of the form \(ax^2+bx+c=0\) in which \(a\neq 0\), \(a\) and \(b\) are coefficients of \(x^2\) and \(x\) respectively and \(c\) is a constant term is called a quadratic equation. Quadratic Equations have one of the three types of solutions –two different values for the variable \(x\), same value for the variable \(x\) or no solution”. This type of introduction with a bit of additional information added or otherwise is observed in many classes.

Obviously the introduction given by the teacher is insufficient. There are concepts learned in other classes which have vertical connection and relevance with the topic quadratic equation, namely algebraic equations, linear equations, factors, constants, coefficients, monomials, binomials and of course, algebraic expressions etc. Sparing 10 to 20 minutes to brush up the memory of the students in the topic is highly essential, if a teacher has to cater to the needs of the students of varying levels of understanding of the subject. Introduction given in a generalized manner without taking into account the students previous knowledge and current knowledge in a topic would serve no purpose.

Working out Problems:

Next the teacher picking up one or two problems randomly from the actual exercise for solving on the black board is a common practice observed in the Mathematics classes. Even selecting the worked out examples given in the text book for black board work is not uncommon among the teachers. While the majority of Mathematics teachers prefer to articulate the steps as they work out on the black board, the rest does not open their mouth while their written work is in progress. In the absence of any instruction, students copy down the black board work without listening much to what the teacher says is a common sight in Mathematics classes. The black board works with our teacher’s explanation cannot be beneficial to the entire class. Leaving a few motivated students, who have developed interest in Mathematics through other sources of learning, the rest would tend to lose interest and accumulate doubts/ignorance over a period of time, if efforts are not made by the teacher to explain the steps as to why and how those steps occur in the way they are written on the black board. If the sequential relation and coherence among the steps in solving a problem is lost sight of, the entire subject matter would present a picture of mystery to students. This results
in aversion to the subject and ultimately mathematics phobia. The brain develops a conditioned response to learning mathematics which I call, ‘Mathematics Blindness’ Anything related to number, order, sequence, logic and Mathematical operations becoming an anathema to the brain.

Handling students’ questions:

The third aspect of teaching Mathematics, namely, how teachers handle questions posed by students, requires a closer examination. Questions, as a matter of fact, are not welcome in mathematics classes. They are perceived as speed breakers to smooth progress (!?) in the completion of syllabus. “After all how good a teacher is not important, but are you a teacher who completes the syllabus within the stipulated period of time is! Where is the time to explain each and everything? Even if you do so, there are not many takers. Parents have more faith in coaching classes than in our ability than to teach their children well. “Explanation likes this fly thick and fast the moment you talk about poor teaching of Mathematics. Even in the best of mathematics classes, there is no guarantee that the skill and the mental process of learning the subject, and the components of mastery learning are taken care of. Moving back and forth in elucidating the concepts of Mathematics is rarely done though it is an essential component to review and refresh the previous knowledge. When teachers proceed without this exercise, students stumble with many a doubt and asking that in the class for clarification is straddled with many a pitfall. Right choice of words for raising questions, receptivity of teacher and the possibility of getting answers from the teacher are all matters of speculation. Hence the students prefer the permissive coaching classes for seeking clarification for the doubts that arise in various levels of learning Mathematics. Yet, their hopes are dashed as coaching classes are as crowded as regular school classes and getting conceptual understanding of the subject becomes a real challenge.

The Home Assignments:

Teachers give large number of exercise problems for the students to solve at home. As seen earlier, doing one or two problems for the name sake does not help the students to acquire the insight required for solving problems at home on their own. The teacher solved problems are inadequate in number and variety, and the explanation, if any, given about problem solving in the classes is either incomprehensive or inadequate. As a result students get frustrated when they struggle with problems with answers not in sight. They lose interest when their woes are not taken care of.

Mathematics-the Queen of All sciences

Mathematics is one of the compulsory subjects of study up to class X and an optional one from class XI onwards. It is an important subject as it is considered as the ‘Queen of all Sciences’. The abstract nature of Mathematics, precision and exactness being its hallmark, makes this subject appears as more difficult than other subjects. Even the simplest of concepts in it like numbers, addition, subtraction, multiplication, division prescribed for the primary classes warrant in-depth understanding and imagination and creative thinking on the part of the teacher for effective teaching. But do we have teachers who possess these qualities in our schools is a moot question.

Teaching Mathematics in the Primary Classes

Teachers are not having the subject specialization in Mathematics too are allotted this subject for teaching in Primary Classes. Concepts in mathematics up to class V level in schools, consists of basic operations such as addition, subtraction etc., that are taught in a routine manner. As a result, the student learns to do those basic operations following certain repetitive patterns oblivious of the “Why” aspect of those patterns. When they reach the middle level (Classes VI to VIII) and later on the secondary level (Classes IX to X), they understand that the ‘patterns ‘that they learn in the primary classes are not of much use and that they need to
know ‘something more’. The real problem to them is to know what is that ‘something more’. It is the understanding of basic concepts which is more essential than knowing certain patterns of doing certain problems in Mathematics. But unfortunately, most students come to middle classes without learning anything about Mathematical concepts and how to use their conceptual understanding for solving problems. Mathematics learning, hence, becomes a big riddle by then, and the slow but steady process of developing disinterest in the subject sets in.

The Challenge in Middle and Secondary Classes:

The teachers of the middle and secondary classes have a challenge on hand: opening up the cognitive domain of students and then taking them to higher order mental abilities though sequential learning process. In other subjects, knowledge and understanding apart, memory play a vital role in scoring marks. Even without the former, with the latter (memory) alone, students can score marks in other subjects, where as in Mathematics, you are expected to do “problem solving” which is a higher order cognitive skill.

The Cognitive Domain:

The cognitive domain of the human brain is said to be responsible for thinking, understanding, imagination and creativity. The cognitive domain becomes a fully functional component of human brain after 10 or 11 years of age in children. This does not mean that this domain remains dormant and nonfunctional before this stage. In fact ‘concept formation’ - one of the difficult functional outputs of the cognitive domain –does take place even before 10 or 11 years of age. The lower order cognitive skills such as knowledge and understanding apart, the elementary level of skills of analysis, and simple problem solving skills are exhibited by primary class children. Systematic development of these skills is called for when the children reach the middle classes. Therefore, a thorough understanding of the process involved in problem solving, which has its genesis in concept learning is a must for teachers of Mathematics.

What Is Concept Learning?

A concept is an abstract idea, and mathematics is full of them. Concept learning involves acquiring a thorough comprehension and grasp of abstract ideas. Each concept in Mathematics has sub-components. For example, ‘algebraic expressions’ is a concept whose sub-components are ‘algebra’ (What?), ‘expression’ (What?), and ‘algebraic expressions’ (definition?). Besides these components, questions like what are numerical expressions, are algebraic expressions different from numeric expressions, what are algebraic equations, how do algebraic expressions differ from algebraic equations, and so on may needs to be answered to bring out clarity in learning the concept ‘algebraic expressions’.

Knowledge Redundancy:

The information age we live in help us see information explosion taking place all around us. The newer learning taking place with geometric progression keeps replacing the current and past information, and hence knowledge is in constant state of flux. Processing information in order to add it to the existing corpus of knowledge is the need of the hour. Teachers, whose main business is transacting knowledge in class room, cannot remain isolated from information processing. They need to keep updating themselves; else they would become knowledge redundant.

Class Room transactions Cognitive Skills:

Knowledge updated by the teachers is to be transacted in an effective manner, in capsules, in class room to facilitate students comprehending it. Students’ language abilities and power of comprehension should be
known to the teacher so as to select the best possible way of communicating knowledge with fosters comprehension. The real task of the teacher wanting to achieve total comprehension exists in analyzing and synthesizing knowledge. This is also acquired to develop application skills - in known situations to start with and progressively in unknown situations. Problem solving requires ‘application skills’, which are the by-product of analysis and synthesis. The skills of analyzing and synthesizing, and application of knowledge at known and unknown situations have an important component called ‘thinking’. Thinking has two integral parts: divergent and convergent, while divergent thinking results in creativity, convergent in conversation.

The vertical Connectivity among the Cognitive Skills:

The skills in various levels of the cognitive domain do not function in isolation. There is a vertical connectivity among them, which can be presented by a flow chart as given under:

**ORDERED SKILLS OF COGNITIVE DOMAIN: THE COGNITIVE LADDER**

![Cognitive Ladder Diagram]

**Information Processing:**

Processing information may require special skills such as skimming and scanning. Yet ‘information’ is kept as the bottom as a lower order skill in view of the fact that the information processed as knowledge is
readily made available in text books to study. Information processing is defined as Claude E. Shannon as the conversion of latent information into manifest information. Latent and manifest information are defined through the terms equivocation (remaining uncertainty, what value the sender has actually chosen), dissipation (uncertainty of the sender what the receiver has actually received) and transformation (saved effort of questioning- equivocation minus dissipation).

**Knowledge:**

Knowledge too has innumerable components yet the bookish knowledge is emphasized in class room teaching and hence its categorization as a lower order skill. Understanding of what is given in text is meant in a limited manner of ‘Knowing what it is’ rather than why and how. The Wikipedia, free encyclopedia, defines knowledge as information of which a person, organization or other entities aware. Knowledge is gained either by experience, learning and perception through association and reasoning. The term knowledge also means the confident understanding of a subject, potential with the ability to use it for specific purpose.

**What is analysis**

There are many definitions given to the term ‘analysis’. Some are given below:

An investigation of the component parts whole and their relations in making up the whole. A form of literary criticism in which the structure of a piece of writing is analyzed. The use of closed-class words instead of infections: e.g., the father of a bride’ instead of ‘the bride’s father’

In our article I use the term ‘analysis’ to refer to understanding the components that go into making something. For analogy, think of a TV set. The components are picture tube, condensers, resistances, speakers etc. are put together to make a composite whole called TV. Similarly any concepts in Mathematics consist of micro-concepts/ sub-concepts, the understanding, defining and elucidating of each micro-concept fall under the domain of analysis. In class VII, for example, the concept of ‘rational number’ is defined as follows:

“Any number that can be put in the form of p/q where p and q are integers and q not equal to zero is a rational number”. Analysis of this concept includes the understanding and elucidation of

i) Why it is said “that can be put in the form of”?
ii) What does ‘any number’ mean?
iii) What are integers?
iv) Why ‘q’ should not be equal to zero?
v) Why and what for is this new set of numbers called rational numbers?
vi) What is ‘rational’ about these rational numbers?

A teacher attempting to teach the definition of rational numbers without throwing the light on the above questions and many more questions related to them is doing disservice to students wanting to learn Mathematics. Given a permissive and receptive atmosphere, students would come out with many questions as given above, the answers of which would be an appetizer for developing their analytical skills.

**Synthesis:**

The word ‘synthesis’ can be defined in many different ways. A few popular definitions are as follows:

- The art of putting different representations together and of grasping what is manifold in them in one act of knowledge.
- Synthesis is what first gives rise to knowledge, i.e. It is not analysis. It is an act of the imagination.
Synthesis suggests the ability to put together separate ideas to form new wholes of a fabric, or establish new relationships.

Synthesis involves putting ideas and knowledge in a new and unique form. This is where innovations truly take place.

The process of bringing pieces of an analysis together to make a whole.

The process of building a new concept, solution, design for a purpose by putting parts together in a logical way.

This is fifth level of Bloom’s taxonomy and deals with the task of putting together parts to form a new whole. This might involve working with parts and putting them together in a creative new way or using old ideas to come up with new ones.

Synthesis is to be done for the purpose of establishing the Gestalt view that ‘the whole is more than the sum total of its parts’. A suitable analogy can be assembling the components of a TV set and making it work.

**Application:**

A few definitions of the ‘application’ are given as follows:

The act of bringing something to bear; using it for a particular purpose: “he advocated the application of statistics to the problem”; “a novel application of electronics to medical diagnosis”

- A diligent effort; “it is a job requiring serious application”
- Utilizing knowledge acquired and processed by the mind for solving problems- both simple and complicated.

The skills associated with information, knowledge, comprehension, analysis and synthesis are to be acquired by students in order to go to the next level of the cognitive order called ‘application’. The application of knowledge, skills, and attitudes has to be done in known situations to start with so that the students can progressively move on to unknown situations. Examples suitably selected can help them go through simple to complex situations and would guide them to acquire insight. This insight is a prime requisite for problem solving.

In school level Mathematics, the skills of analysis and synthesis and the insight learning that takes place as a result of the application of those skills would pave the way for solving exercise problems, which the teachers shy away from under the pretext of lack of time, and other priorities. The skills in the cognitive ladder are vertically connected, and the acquisition of those skills at each level requires the student’s to allow their minds to think and assimilate ideas. This repetitive manner in which the sequential cognitive skills practiced would train the mind in Mathematical thinking which is otherwise called logical thinking.

**Logical Thinking:**

Logical thinking is defined as that thinking which is coherent and rational. Reasoning and abstract thought are synonymous with logical thinking. Logical thinking be in Mathematics or any other subject is required to establish the coherence of facts of matter and formation of logical patterns and sequences. Mind has the special ability to think and assimilate, and retain subject matter when presented in sequential manner. Mind grasps matter devoid of gaps quickly. Unanalyzed knowledge in its un-synthesized form poses difficulty in retaining it in long time memory, as concepts and its components do not function isolation. Hence, mind rejects fragmented information which lacks patterns.
**Problem Solving:**
The thought process involved in solving a problem is called problem-solving. Problem solving as a skill is developed crossing various other skills on its way. The skills lying down below ‘problem solving’ in the cognitive ladder can be compared to the floors of a building. You cannot reach the sixth floor without crossing the floors down below. Similarly when problem solving is attempted in classes with making explicit efforts to pass through the levels of knowledge, understanding, analysis, synthesis, and application, students fail miserably.

Often it is said that practice and drill in Mathematics would help learn the subject better. Hence again, by repeatedly working out problems, one has to ‘memorize’ the steps, but it does not guarantee success when problems are differently worded or twisted. Following the cognitive order-moving from information to problem solving steps in the classroom will help the students know the sequential mental processes involved in solving problems in Mathematics. As they practice these steps regularly it will boost their confidence in learning the subject. But classroom learning these days mostly concentrate on problem solving as a direct hit strategy. Working out the problems first without following the cognitive order is equivalent to putting the horse behind the cart, which will take the students nowhere. Even conscientious teachers tend to spend 10-20% of their class time on concept teaching and 80-90% on working out the problems. This is totally incorrect. Concept learning and concept formation require knowledge comprehension, analysis and synthesis. After going through these steps, as the next stage, ‘application’ should be dealt with. At the end comes problem solving. The process of going through and gaining thorough grasp of knowledge, understanding, analysis, and synthesis warrants 80-90% of class room time, and hence just 10-20% class time is enough for problem solving.

**The Cognitive Order Learning:**
Failure to recognize orderly thinking, the basic quality of cognitive order, is the main reason for the difficulty faced by students in learning Mathematics. The earlier the teachers and students recognize the need for approaching Mathematics logically, the better it would be to create and sustain interest in the subject. The Mathematics classroom practices may, therefore, be fashioned by following the sequential steps given as under:

i) Teacher utilizing 5-10 minutes in the beginning of the class on asking questions on the knowledge, comprehension, analysis and synthesis part of the chapter on hand.

ii) After identifying the skill area in which students have problems, discussion to thrash out those problems should be taken up. Often the problems of students stem from lack of understanding of the basic concepts. It is essential therefore to keep doing ‘concept recall’ and ‘concept clarification’. Comprehending basic concepts is an essential condition for moving on the further steps in the cognitive order, namely, analysis, synthesis etc.

iii) Then the points under application of skills down below may be taken up for discussion. Once the gray areas in application are cleared, the students may be instructed to do problem solving.

iv) If students falter in steps, the logical sequence of steps followed to solve any problem in the given chapter falling under the concepts learnt may be discussed again.

v) Effective questioning to draw out the conceptual understanding of the subject matter learnt by the students should be done at least every tenth minute in every period to ensure that they are actually with the teacher.

vi) Free-wheeling of ideas related to the subject by the students should be encouraged as it would help throw new light on the subject matter under study.

vii) Questions by the students, however silly they may seem, should be welcome in the class and the teacher should listen to them with patience and convincing answers given.
viii) After a full-fledged concept learning session following cognitive order learning, problem solving should be taken up where the students should be encouraged to work out the problems under the watchful eyes of the teacher.

ix) Vertical and horizontal connectivity of concepts in mathematics should always form an integral part of teaching learning, and students being thorough in the sequential conceptual elements be taken care of.

**The Critical aspects of learning Mathematics:**

Besides taking care of the above nine aspects of teaching, teachers desirous of making students love and do well in mathematics should need to pay heed to the following aspects as well:

A thorough comprehension of the domains - psychological, physical and practical - of effective learning of the subject by the students;

The process they have to follow scrupulously in acquiring skills for the mastery-level learning of the subject;

The role the teachers and parents have to play in fostering and sustaining students interest and enthusiasm so that they learn the subject with ease at classroom and face the examination with confidence.

Let me sum up some of the benefits of Cognitive-order-Learning: This methodology- ‘Cognitive-order-Learning’ enables the students -

- To acquire subject learning competencies
- To develop problem solving skills
- To boost their confidence in the subject
- To widen their interest in the areas of mathematics
- To have and sustain self-directed and self-motivated activities in mathematical learning.
- To achieve mastery level learning of the subject.
- To help apply the skills acquired in Mathematics to other subjects.
- To utilize the cognitive domain to its full extent
- To remove examination phobia.

**Conclusion:**

The subject matter of Mathematics teachers is vast. An attempt has been made to give only the most rudimentary aspects of it. What requires as a clear understanding on the part of the teachers is that the subject, Mathematics, is neither difficult unconquerable. Yet, it is perceived to be so mostly owing to ineffective teaching, which jumps from knowledge to problem solving, leaving a vast territory of skills in between untouched. As said earlier, Mathematics being the queen of all science deserves an approach to teaching which is based on the sound scientific principles of human learning. In any class room, if students declare that they like Mathematics they enjoy learning it, and they have no difficulty in solving the problems, that class is said to have been blessed with a teacher teaching Mathematics the way it is deserves to be taught. That way surely is Cognitive-order-Learning approach with the thorough understanding of the critical aspects of learning Mathematics referred to above.

**Dr. E.T. Arasu, Dy. Commissioner &
Director K.V. ZIET Mysore.**
NEW TRENDS IN ASSESSMENTS

Introduction:

One of the main reasons for teachers to assess student learning is to obtain feedback that will guide teaching and assist in making modifications to lesson planning and delivery to ensure student progress. Assessment allows teachers to monitor progress, diagnose individual or group difficulties and adjust teaching practices. Assessment can support student motivation when students are provided with on-going information about their progress and with opportunities to set further goals for learning. Assessment is an interactive process between students and faculty that informs faculty how well their students are learning what they are teaching. The information is used by faculty to make changes in the learning environment, and is shared with students to assist them in improving their learning and study habits. This information is learner-centred, course based, frequently anonymous, and not graded.

Current trends in classroom Assessment:

The terms formative assessment and summative assessment are being redefined in education circles. Many teachers know formative assessment as the informal, daily type of assessment they use with students while learning is occurring. Summative assessment was the term used to “sum it all up,” to indicate a final standing at the end of a unit or a course. Current trends in assessment focus on judging student progress in three ways: Assessment for learning, assessment as learning and assessment of learning. Each assessment approach serves a different purpose.

Assessment for learning is especially useful for teachers as they develop, modify and differentiate teaching and learning activities. It is continuous and sustained throughout the learning process and indicates to students their progress and growth.

In assessment for learning, teachers monitor the progress made by each student in relation to the program of studies, outcomes and determine upcoming learning needs. Teachers ensure that learning outcomes are clear, detailed and ensure that they assess according to these outcomes.

They use a range of methods to gather and to provide students with descriptive feedback to further student learning. These methods may include checklists and written notes based on observations of students as they learn. The descriptive feedback gathered is used to inform planning for learning and to assist the teacher in differentiating instruction in order to meet the needs of all students.

The feedback may be shared in oral or written form with individual students or with the class as a whole. As the information gathered guides the planning process, it leads to the improvement of future student performance in relation to specific outcomes.

Assessment as learning focuses on fostering and supporting metacognitive development in students as they learn to monitor and reflect upon their own learning and to use the information gathered to support and direct new learning.

It focuses on the role student’s play in their learning. In this approach to assessment, students are viewed as the bridge between what they know and the unknown that is still to be learned. Their role is to assess critically both what and how they are learning. They learn to monitor their thinking and learning processes; to understand how they are acquiring and retaining new information or developing new skills and awareness; and how to make adjustments, adaptations and even changes when necessary.
For some students, being asked to reflect on their learning by using skills and strategies related to metacognition (to think about thinking) might seem new and uncomfortable. They may need help to come to the realization that learning is a conscious process in which knowledge is constructed when the known, or previously acquired, encounters the new or unknown. This process often results in the restructuring or reintegration of what was previously learned.

**Assessment of learning** is cumulative in nature. It is used to confirm what students already know and what they can do in relation to the program of studies outcomes. Student progress is reported by way of a mark; e.g., a percentage or letter grade, a few times a year or a term. The report card is usually received by students, their parents/guardians as well as by school administrators.

Assessment of learning takes place at specific times in the instructional sequence, such as at the end of a series of lessons, at the end of a unit or at the end of the school year. Its purpose is to determine the degree of success students have had in attaining the program outcomes. Assessment of learning involves more than just quizzes and tests. It should allow students to move beyond recall to a demonstration of the complexities of their understanding and their ability to use the language.

Assessment of learning refers to strategies designed to confirm what students know, demonstrate whether or not they have met curriculum outcomes or the goals of their individualized programs, or to certify proficiency and make decisions about students’ future program or placements.

**Teacher reflections Assessment procedures:**

It is important for a teacher to reflect on why and when students’ progress is assessed.

The types of reflective questions that teachers can ask themselves when engaged in assessment for learning include:

► Am I observing in order to find out what my students know or are able to do?

► Does my assessment strategy allow student learning to be apparent? Are there elements I need to change in order to minimize anxiety or distractions that might get in the way of learning?

► Will I use the results of my observations to modify my instruction, either with a particular student or with a group of students, or the next time I teach this concept or skill to a new class?

► Will I share the results of my observations with the individual student so that the student and I can decide how to improve future performance?

► Will I share the results of my observations with the class in general (without identifying particular students) in order to provide some indicators as to where they can improve future performance?

The types of reflective questions that teachers can ask themselves when planning opportunities in support of assessment as learning include:

► Are the students familiar with the purpose of reflective tools, such as the one I am thinking of using? Will they be able to engage with the questions in a meaningful way?

► Have I provided/will I provide support for students in accordance with the various points mentioned in the reflective instrument; i.e., do I provide clear instructions, create a model, share a checklist, ensure that there are reference materials?
Teacher reflections: The types of reflective questions that teachers can ask themselves when planning opportunities in support of assessment include:

► Am I using processes and assessment instruments that allow students to demonstrate fully their competence and skill?

► Do these assessments align with the manner in which students were taught the material?

► Do these assessments allow students to demonstrate their knowledge and skills as per the program of studies outcomes?

**Student reflection assessment (Assessment as learning):**

Students record their reflections by completing sentence starters such as “Things that went well …”; “Things that got in my way …”; “Next time I will ….”

Alternatively, they may check off various statements that apply to themselves or their performance on a checklist.

An overview of the different practices and variety of instruments that can be used and tailored to meet the needs of a specific assessment purpose.

<table>
<thead>
<tr>
<th>Assessment for Learning</th>
<th>Assessment as Learning</th>
<th>Assessment of Learning</th>
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</thead>
<tbody>
<tr>
<td>Informal observation/Formative assessments/Peer learning</td>
<td>Conferencing/Learning conversations/Peer assessment/Quizzes or Tests/ Self-assessment and Goal setting.</td>
<td>Performance Tasks/Projects Summative assessment Quizzes/Pen-paper tests Tests or Examinations. PSA</td>
</tr>
</tbody>
</table>

**Formative Assessment** is a process used by teachers and students as part of instruction that provides feedback to adjust ongoing teaching and learning to improve students’ achievement of core content. As assessment for learning, formative assessment practices provide students with clear learning targets, examples and models of strong and weak work, regular descriptive feedback, and the ability to self-assess, track learning, and set goal. Formative assessments are most effective when they are done frequently and the information is used to effect immediate adjustments in the day-to-day operations of the course.

Assessment is not formative unless something is “formed” as a result of interpreting evidence elicited. It informs teacher where the need/problem lies to focus on problem area. It helps teacher give specific feedback, provide relevant support and plan the next step. It helps student identify the problem areas, provides feedback and support. It helps to improve performance and provides opportunity to improve performance. Peer learning can be encouraged at all stages with variety of tools. Formative Assessment Strategies:
## Tools for Formative Assessment

<table>
<thead>
<tr>
<th>Technique</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>One minute answer</td>
<td>A one-minute answer question is a focused question with a specific goal that can be answered within a minute or two.</td>
</tr>
</tbody>
</table>
| Analogy prompt             | A designated concept, principle, or process is like ______ because_______________.
| Think, pair, share/Turn to your partner | Students think individually, then pair (discuss with partner), then share with the class. |
| 10-2 theory/35-5 theory    | 10 minutes instruction and two minutes reflection/35 minutes instruction and 5 minutes reflection. |
| Self-assessment            | A process in which students collect information about their own learning, analyze what it reveals about their progress toward intended learning goals or learning activity or at the end of the day. |

### Conclusion:

Teachers should continuously use a variety of tools understanding different learning styles and abilities and share the assessment criteria with the students. Allow peer and self-assessment. Share learning outcomes and assessment expectations with students. Incorporate student self-assessment and keep a record of their progress and Teachers keep records of student progress.
TEACHING OF MATHEMATICS – MOVING FROM MATHPHOBIA TO MATHPHILIA

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“Mathematics is for everyone and all can learn Mathematics” - NCF 2005

The Little Oxford Dictionary define phobia as fear or aversion. Psychology textbooks describe it as an abnormal fear. We hear of claustrophobia, acrophobia, nyctophobia, and anthropophobia.

The pioneers in the study of Mathematics anxiety, Richardson and Suinn (1972), defined Mathematics anxiety in terms of the (debilitating) effect of mathematics anxiety on performance: “feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations”.

Is there such a thing as math phobia? To know the answer one needs to only teach mathematics particularly in the secondary and senior secondary classes. And the reality is that most school drop outs in the Board exam are due to failure in Mathematics. Studies indicate that students’ anxiety about Mathematics increases between the sixth and twelfth grade.

With this reality check, this write-up aims to analyse the problem and by this parsing, redefine the teaching learning of Mathematics firmly grounded on foundations of success- for the student for the teacher, the society and the nation.

The suggestion that Mathematics anxiety threatens both performance and participation in Mathematics, together with the indications that Mathematics anxiety may be a fairly widespread phenomenon (e.g. Buxton, 1981), makes a discussion like this, concerning Mathematics anxiety in students, particularly the Board going students, of extreme importance.

Mathematics is termed as the queen of all sciences, having logical thinking as its crown and problem solving as its sceptre. Two essential elements which are necessary not just to master nuances of the numeral world but more importantly to have success in life in qualitative ways- these two are also the core life skills formulated by WHO for a healthy and successful life.

The question is, “Does the teaching of Mathematics in our classrooms realise any of these objectives? The huge population of children who balk at the very mention of the subject is an ever growing one as generation gives way to another.

The NFG Position paper on the teaching of Mathematics under the section “Problems in Teaching and Learning of Mathematics” states: four problems which we deem to be the core areas of concern:

Other problems are systemic in nature:

Compartmentalisation- Segregation of Primary, Secondary and Senior secondary

Curricular acceleration- The quantum and scope of the syllabus is much larger and wider with passing days.

The NFG recommends four fold measures to ensure that all children learn Mathematics:
Shifting the focus of mathematics education from achieving ‘narrow’ goals to ‘higher’ goals, whole range of processes here: formal problem solving, use of heuristics, estimation and approximation, optimization, use of patterns, visualisation, representation, reasoning and proof, making connections, mathematical communication. Giving importance to these processes constitutes the difference between doing mathematics and swallowing mathematics, between mathematisation of thinking and memorising formulas, between trivial mathematics and important mathematics, between working towards the narrow aims and addressing the higher aims.

1. **A sense of fear and failure regarding mathematics among a majority of children** *(cumulative nature of mathematics, gender and social biases about math, use of language and more importantly symbolic language)*

2. **A curriculum that disappoints both a talented minority as well as the non-participating majority at the same time** *(emphasises procedure and knowledge of formulas over understanding is bound to enhance anxiety)*

3. **Crude methods of assessment that encourage perception of mathematics as mechanical computation, and** *(only one right answer, sacrificing the process for the right solution, overemphasis on computation and absolute neglect for development of mathematical concepts)*

4. **Lack of teacher preparation and support in the teaching of mathematics** *(outdated methodology, depending on commercial guides due to insufficiency in conceptual clarity and understanding of the fundamentals of mathematics, inability to link formal mathematics with experiential learning, particularly in the secondary and senior secondary stages, incapacity to offer connections within mathematics or across subject areas to applications in the sciences)*

The recommended methods are:

Cross curricular and integrated approaches within mathematics and across other disciplines,

Simplifying mathematical communication multiplicity of approaches, procedures, solutions using the common man’s mathematics or “folk algorithm”- basing problems on authentic real/daily life contexts use of technology

1. **Engaging every student with a sense of success**, while at the same time offering conceptual challenges to the emerging mathematician- striving to reduce social barriers and gender stereotypes and focussing on active inclusion of all children in the teaching-learning of mathematics. Children with math phobia usually seem to have little confidence in themselves. They feel they are not good in math; they refrain from asking questions (little realizing that more than half the class is puzzled over the same Problem!); they are afraid to answer any question directed to them for fear of being labelled "dumb" or "stupid." Such fear or anxiety about math often begins during the Primary years and continues through life.
Recommendations:

✔ Teacher need to model “problem –solving” particularly in the context of word problems. To work out diverse problems and build personal repertoire of problem solving skills and model them with enthusiasm and confidence.

✔ Move from simple step problem solving modes to increasingly complex and multi-step problem solving.

✔ Inculcate positive, persevering problem solving approaches- solve problems with them building rapport thus building their self-esteem and confidence.

✔ Use a “problem solving” bulletin board to bring problem solving as part of everyday learning activity.

✔ In problem solving, arriving at the "correct answer" is not the most important step. More important is choosing the correct strategy for solving the problem. Even though there is only one correct answer, there will be more than a single correct strategy for solving a problem. When students are reassured of this fact, they will then be more willing to tackle new problems.

2. **Changing modes of assessment** to examine students’ Mathematisation abilities rather than procedural knowledge.

3. **Enriching teachers with a variety of Mathematical resources.** - The development of teacher knowledge is greatly enhanced by efforts within the wider educational community. Teachers need the support of others—particularly material, systems, and human and emotional support. While teachers can learn a great deal by working together with a group of supportive mathematics colleagues, professional development initiatives are often a necessary catalyst for major change. Activities like collaborative and strategic approaches, Mathematics Lab and experiments help in this aspect.

Reflecting on and applying these thoughts to the KV context, what should the maths teachers need to do to ensure that all students learn Mathematics in the true sense of the word i.e. love it, think, learn and apply it.

Mathematics teachers need to move from emphasis on Computation to holistic Mathematical concept learning which will mathematise their thoughts and perspectives.

They need to be constantly conscious of and strive to promote a sense of achievement and comfort in learning of mathematics.

CONCLUSION

UNESCO’s The International Academy of Education in its paper-Effective Educational Practices Series on the topic” Effective Pedagogy in mathematics” by Glenda Anthony and Margaret Walshaw postulates the following:

1. **An Ethic of Care**- Caring classroom communities that are focused on mathematical goals help develop students’ mathematical identities and proficiencies.

2. **Arranging For Learning**- Effective teachers provide students with opportunities to work both independently and collaboratively to make sense of ideas.

3. **Building on Students’ Thinking**- Effective teachers plan mathematics learning experiences that enable students to build on their existing proficiencies, interests, and experiences.

4. **Worthwhile Mathematical Tasks**- Effective teachers understand that the tasks and examples they select influence how students come to view, develop, use, and make sense of mathematics.
5. **Making Connections** Effective teachers support students in creating connections between different ways of solving problems, between mathematical representations and topics, and between mathematics and everyday experiences.

6. **Assessment for Learning**- Effective teachers use a range of assessment, practices to make students’ thinking visible and to support students’ learning.

7. **Mathematical Communication**- Effective teachers are able to facilitate classroom dialogue that is focused on mathematical argumentation

8. **Mathematical Language**- Effective teachers shape mathematical language by modelling appropriate terms and communicating their meaning in ways that students understand

9. **Tools And Representations**- Effective teachers carefully select tools and representations (number system itself, algebraic symbolism, graphs, diagrams, models, equations, notations, images, analogies, metaphors, stories, textbooks and technology) to provide support for students’ thinking

10. **Teacher Knowledge**- Teacher content knowledge, Teacher pedagogical content knowledge

The referred UNESCO paper can be downloaded from the websites of the IEA (http://www.iaoed.org) or of the IBE (http://www.ibe.unesco.org/publications.htm)
Qualities of a Successful Mathematics Teacher

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A teacher who is attempting to teach without inspiring the pupil with a desire to learn is hammering on a cold iron ---Horace Mann

Not all students like mathematics, but a good mathematics teacher has the power to change that. A good mathematics teacher can help students who have traditionally struggled with mathematics begin to build confidence in their skills. Successful mathematics teachers have certain qualities that make them the experts they are. These are the teachers required by the society, because of their knowledge, style and handle on the subject; they know what really work for students.

A good mathematics teacher can be thought to need some qualities that are connected to his view of mathematics. This view consists of knowledge, beliefs, conceptions, attitudes and emotions. Beliefs and attitudes are formed on the basis of knowledge and emotions and they influence students' reactions to learn future Mathematics

- A good mathematics teacher should have sufficient knowledge and love of mathematics. He needs to have a profound understanding of basic mathematics and to be able to perceive connections between different concepts and fields.

- A teacher should have a sufficient knowledge of mathematics teaching and learning. He needs to understand children' thinking in order to be able to arrange meaningful learning situations. It is important that the teacher be aware of children' possible misconceptions. In addition, he needs to be able to use different strategies to promote children’ conceptual understanding.

- A good mathematics teacher also needs additional pedagogical knowledge: the ability to arrange successful learning situations (for example, the ability to use group work in an effective way), knowledge of the context of teaching and knowledge of the goals of education.

- A good mathematics teacher's beliefs and conceptions should be as many-sided as possible and be based on a constructivist view of teaching and learning Mathematics.

- In the classroom, a talented mathematics teacher serves as a facilitator of learning, providing students with the knowledge and tools to solve problems and then encouraging students to solve them on their own. When students answer a problem incorrectly, he does not allow them to quit. He encourages students to figure out where they went wrong and to keep working at the problem until they get the correct answer, providing support and guidance where needed.

- A Good Mathematics teacher should have the ability to do quick error analysis, and must be able to concisely articulate what a student is doing wrong, so they can fix it. This is the trickiest part of being a good Mathematics Teacher. He should have ability to assign the home work that targeted what the student is learning in the classroom to minimize the mistakes committed and to have proper practice on the concepts taught.

- A successful Mathematics Teacher is seen as a leader in his classroom and in the school. His students respect him, not only for his knowledge of Mathematics, but for his overall attitude and actions. Students can tell that he respects them as well. He has control over the classroom, laying out clear rules and expectations for students to follow.
• A good mathematics teacher focuses less on the content being taught than the students being taught. A good mathematics teacher cares about his students and recognizes when a student needs some encouragement and addresses the problem to help the student refocus on the content.

• A Good mathematics teacher, in particular, possesses enormous amount of patience, because there are many different ways that students actually learn mathematics. And they learn at many different speeds. Math teachers are not frustrated by this attitude of students. He should have sufficient understanding Jean Piaget’s theory on how youngsters create logic and number concepts.

• A Good mathematics teacher never lives in the past. He knows how to unlearn outmoded algorithms and outdated mathematical terms and re-learns new ones. He appreciates the change with all enthusiasm and welcomes it.

• He is approachable and explains, demonstrates new concepts/problems in detail and creates fun. He commands respect and love by his subject knowledge and transaction skills.

• A good teacher sets high expectations for all his students. He expects that all students can and will achieve in his classroom. He doesn’t give up on underachievers.

• A great teacher has clear, written-out objectives. Effective teacher has lesson plans that give students a clear idea of what they will be learning, what the assignments are and what the promoting policy is. Assignments have learning goals and give students ample opportunity to practice new skills. The teacher is consistent in grading and returns work in a timely manner.

• Successful teacher is prepared and organized. He is in his classrooms early and ready to teach. He presents lessons in a clear and structured way. His classes are organized in such a way as to minimize distractions.

• Successful teacher engages students and get them to look at issues in a variety of ways. He uses facts as a starting point, not an end point; he asks "why" questions, looks at all sides and encourages students to predict what will happen next. He asks questions frequently to make sure students are following along. He tries to engage the whole class, and he doesn’t allow a few students to dominate the class. He keeps students motivated with varied, lively approaches.

• A good Mathematics teacher forms strong relationships with his students and show that he cares about them. He is warm, accessible, enthusiastic and caring. Teacher with these qualities is known to stay after school and make himself available to students and parents who need his services. He is involved in school-wide committees and activities and demonstrates a commitment to the school.

• A good mathematics teacher communicates frequently with parents. He reaches parents through conferences and frequent written reports home. He doesn't hesitate to pick up the telephone to call a parent if he is concerned about a student.

• There are five essential characteristics of effective mathematics lessons: the introduction, development of the concept or skill, guided practice, summary, and independent practice. There are many ways to implement these five characteristics, and specific instructional decisions will vary depending on the needs of the students. The successful mathematics teacher should have these characteristics in his regular teaching practice.

• In addition, every good Mathematics teacher has the positive values like Accuracy, Alertness, Courtesy, Empathy, Flexibility, Friendliness, Honesty, Initiative, Kindness, Loyalty, Patience, Responsibility, Stability, Tactfulness and Tolerance.
“The mathematics teacher is expected to have proficiency in the methodology ‘cognitive –order – learning’ which enables the students to acquire subject learning competencies, to develop problem solving skills, to boost their confidence in the subject, to widen their interest in the areas of Mathematics, to have and sustain self-directed and self-motivated activities in mathematics learning, to achieve mastery level learning of the subject, to help apply the skills acquired in learning mathematics to other subjects, to utilize the cognitive domain to its fullest extent and to remove examination phobia”.

National Curriculum Framework – 2005 envisages that

- The main goal of mathematics teacher in teaching Mathematics should be Mathematisation (ability to think logically, formulate and handle abstractions) rather than 'knowledge' of mathematics (formal and mechanical procedures)

- The Mathematics teacher should have ability to teach Mathematics in such a way to enhance children’ ability to think and reason, to visualize and handle abstractions, to formulate and solve problems. Access to quality mathematics education is the right of every child.

A balanced, comprehensive, and rigorous curriculum is a necessary component for student success in mathematics. A quality mathematics program which includes best mathematical tasks and models to assist teachers is essential in making sound instructional decisions that advance student learning.

**********
TEACHING STRATEGIES IN MATHEMATICS FOR EFFECTIVE LEARNING

Mr. Siby Sebastian
Principal, K V Bijapur (Karnataka)

Mathematics by virtue of its boundless practical applications and tasteful bid of its methods and results has long held a prominent place in human life. From the quick arithmetic that we do in our everyday lives to the onerous calculations of science and technology, Mathematics shapes and effects about every item around us.

But for many secondary and senior secondary students, Mathematics consists of facts in a vacuum, to be memorized because the teacher says so, and to be forgotten when the course of study is completed. In this common scenario, young learners often miss the chance to develop skills—specifically, reasoning skills—that can serve them for a lifetime.

In my 20+ years of mathematics teaching in schools across our country and in foreign lands, I have seen some truly remarkable changes in the way secondary school children perceive Mathematics and their ability to succeed in it depend upon the pedagogy.

Discovering approaches to make Mathematics exciting for students who are in the middle of the pack could have a profound effect on their futures. It would attract many students who are apprehensive in their own abilities into advanced careers. But it is going to require a fundamentally different approach to teaching mathematics from childhood through secondary school. Here are a few of the many possible ideas to begin that change.

Recreational Mathematics

Recreational inspiration consists of puzzles, games or contradictions. In addition to being selected for their specific motivational gain, these procedures must be brief and simple. An effective implementation of this procedure will allow students to complete the "recreation" without much effort. Using games and puzzles can make Mathematics classes very amusing, exciting and stimulating. Mathematical games provide opportunities for students to be dynamically involved in learning. Games allow students to experience success and satisfaction, thereby building their enthusiasm and self-confidence. But Mathematical games are not simply about fun and confidence building. Games help students to: understand Mathematical concepts, develop Mathematical skills, know mathematical facts, learn the language and vocabulary of Mathematics and develop ability in mental Mathematics.

Investigating Mathematics

Many teachers show students how to do some problems and then ask them to practice. Teachers can set students a challenge which hints them to discover and practice some new problems for themselves. The job for the teacher is to find the right challenges for students. The challenges need to be matched to the ability of the learners. The key point about investigations is that students are stimulated to make their own decisions about; where to start, how to deal with the challenge, what Mathematics they need to use, how they can communicate this Mathematics and how to describe what they have discovered. We can say that investigations are open because they leave many choices open to the student.

Creativity in Mathematics

Creativity is a word that is perhaps more easily associated with art, design and writing than it is with Mathematics, but this is wrong. Mathematics requires as much creativity in its teaching and learning as any
other subject in the curriculum. It is important to remember that creative teaching and learning not only needs teachers to use creativity in planning inventive and thought provoking learning opportunities but must also encourage creative thinking and response from learners. A lesson in which the teachers’ delivery and resources are creatively delivered but which fails to elicit creative thinking and response from students has not been fully successfully creative lesson.

Problem solving is a key to Mathematics and this in itself presents an excellent way of encouraging creativity in your lessons. It is a common belief that a degree of rote learning is necessary before learners can engage in problem solving, but such an attitude may have the effect of pre-empting genuine creative thinking.

**Group work**

Research evidences has consistently shown that, regardless of the subject being studied learners working together in small groups tend to make greater progress in learning what is taught than when the same content is taught in other more didactic ways. Learners working collaboratively also appear more satisfied with their classes and have been shown to have greater recollection of learning. There are numerous ways in which you can arrange learners into groups in your class room. Informal groups’ can be created by asking learners to turn to a neighbour and spend 2-3 minutes discussing a question you have posed. Such informal group can be arranged at any time in a class of any size to check on learners understanding, to provide an opportunity to apply new knowledge, or to provide change of pace within the lesson. A more formal arrangement can be made by the teacher establishing the groups. There are conflicting ideas for this but my personal preference is always for mixed ability group.

**ICT in Mathematics Teaching and Learning**

Appropriate use of ICT can enhance the teaching and learning of Mathematics in secondary and senior secondary level. ICT offers powerful opportunities for learners to explore Mathematical ideas, to generalize, explain results and analyse situations, and to receive fast and reliable, and non-judgemental, feedback. Their use needs careful planning – not just showing a power point presentation but also of activities that allow for off-computer Mathematical thinking as well as on-computer exploration. Decisions about when and how ICT should be used to help teach mathematical facts, skills or concepts should be based on whether or not the ICT supports effective teaching of the lesson objectives. The use of ICT should allow the teacher or learners to do something that would be more difficult without it, or to learn something more effectively or efficiently.

**Theatre in Mathematics**

The individuals who had the delight of being in front of an audience or performing in any capacity before an audience needn’t be convinced about the magic of theatre. The world of theatre is one of the most important ways children learn about actions and implications, about customs and dogmas, about others and themselves. Students in every class room can claim the supremacy and potential of theatre today. We don’t have to wait for costly tools and amenities. An occasion to create their own dramas based on what they learned in math and backing them in implementation improve the communication, leadership and motivation skills which will have a long lasting effect in their memory.

The National Focus Group Position Papers on all segments linking to education are an extraordinary repository of ideas, theory and procedure for teachers. The position paper devoted to Arts, music, dance and theatre clearly mentions why and how it may be integrated in the classroom and invokes what is called “Sensitivity Pyramid through Drama”. In cognizance with the NFG position papers theatre can work...
extremely well as micro level experimental innovative and creative math pedagogy. Theatre is an effective learning tool as it deals with action and imagination, understanding the concept being taught with a view to applying this understanding to real life situations.

*Function dance performed by class XI students*
TEACHING LEARNING MATHEMATICS WITH JOY

Mrs. Sharada. M
Teacher, DMS, RIE Mysore

Every child is naturally motivated to learn and are capable of learning. Children construct knowledge by connecting the existing ideas with the new ideas. The teaching of Mathematics must enable them to examine and analyze their everyday experience. Mathematics is the pivot of analytical and rational thinking. It requires a constant practices to retain different methods, theories, proofs and reasons in memory. It can be done only through a systematic approach. The syllabus in the subject of Mathematics has undergone changes from time to time in accordance with the changing needs of the society. The curriculum at the secondary and senior secondary stage primarily aims at enhancing capacity of students to apply mathematics in solving day-to-day life problems and students should acquire the ability to solve problems.

The NCF-2005 (National curriculum frame work-2205) has elaborated on the insight of learning without burden to ensure that a child is not taken away from the joy of being young by de-linking school knowledge from everyday experience. One of the most important areas in this respect is regarding mathematics learning in schools. It is a common observations that a large number of students consider mathematics as a difficult subject when they enter secondary/senior secondary level. This is creating a phobia in the minds of students towards mathematics. This misconception makes the subject move abstract at that level. It is mainly due to wrong teaching practices which do not link the subject with their real life. It is very essential to know and make them understand that mathematics is very much related to real life, instead of teaching the subject in a mechanical manner where students are made to memorize formulae, theorems, proofs, algorithms etc. and apply these in solving problems.

It is in the hands of teachers to make mathematics teaching learning process of joyful experience for the learner. For this purpose a teacher has to make use of varieties of activities which involves student’s participation in the development of concepts. Creating link between within the subjects and across the subjects motivates children and helps them to appreciate the subject. Mathematics has been projected as an abstract subject much to be feared by students of limited capabilities. The teachers can drive this phobia and make them understand the importance of the subject, that it has application in almost all walks of life and also through mathematics we can describe – understand and work with physical phenomena with utmost precision.

Teaching is a noble profession and the teachers are the one who has to protect novelty of this profession. The teachers’ positive attitude and commitment towards the profession will certainly motivate a child to learn better mathematics need to be taught in an interactive manner by involving children in the teaching-learning process. The theories can be developed by asking questions and using examples and illustrations based on their daily life situation. This promotes independent thinking and problem solving skills in children. Teacher is a constant learner and a facilitator in the teaching-learning process. Gone are those days when teachers were to teach and children were there to listen and learn. In the present context- their knowledge and ability to transact the curriculum in the manner the children want to learn. For this the teachers have to improve the pedagogic skills. Every teacher will have to improve their teaching as well as evaluation techniques in order to ensure students to learn mathematics and love mathematics.

Let us think for a while and try to get a proper answer for the question “what makes mathematics so difficult and fearsome for many students?” It may be because of the subject itself or because of the person who teaches the subject. Ultimately, it is the teacher who has to make the subject interesting to learn and enable the child to understand the importance of it in one’s life. So make use of latest technology support activities,
real life situations and constructivist approach in the process of teaching learning. Introduce new concepts in a simple language, keeping in mind the language ability of children. Basic concepts have to be explained through attractive illustrations which connects them to their life outside the classroom.

Training programmes and workshops for creating appropriate leaning materials have been very helpful to the teachers in recent days in creating a classroom with difference. To meet the challenges of today and further it is necessary for a teacher to work with open-mind towards the learning situations. A resource material prepared by the experienced teachers would certainly help teachers to teach with ease, get a lot of ideas to transact and make the evaluation continuous and comprehensive. So teachers can make use of such opportunities and can bring name and fame to teaching profession.

Welcome! Let’s enjoy the teaching profession with our students.
Resources – Chapter wise

TERM -1
CHAPTER-1

REAL NUMBERS

INTRODUCTION

In this chapter, we begin with Euclid’s division lemma, Euclid’s division algorithm and the Fundamental theorem of Arithmetic. Euclid’s division lemma tells about divisibility of integers. Euclid’s division algorithm provides us a stepwise procedure to compute the HCF of two integers. The Fundamental theorem of Arithmetic tells us about expressing positive integers as the product of prime integers. We apply this theorem to prove the irrationality of many numbers such as \(\sqrt{2}\), \(\sqrt{3}\), \(\sqrt{5}\), etc. We know that the decimal representation of a rational number is either terminating or non terminating repeating. The prime factorisation of the denominator of a rational number completely reveals the nature of its decimal representation.

EXPECTED LEARNING OUTCOMES

1. To revisit number system from Natural number to Real Numbers.
2. To recall Euclid’s division lemma and Euclid’s division algorithm
3. To apply Euclid’s division algorithm to calculate HCF of two or three positive integers.
4. To understand the fundamental theorem of Arithmetic and to find HCF and LCM of two or three numbers.
5. Verifying when a number expressed in the exponential form can end with digit zero (eg: can \(6^n\) end with digit 0).
6. To understand, for any two positive integers a, b
   \[
   \text{HCF} (a, b) \times \text{LCM} (a,b) = a \times b.
   \]
7. To understand that if p is prime number and p divides \(a^2\), \(a > 0\) then p divides a.
8. To prove irrationality of numbers - Proof by contradiction.
9. To revisit decimal expansion of real numbers
10. To verify that the decimal expansion of every rational number is terminating or Non-terminating repeating.


**GRADED EXERCISE**

**LEVEL-1**

(1Mark)

1. Use Euclid’s division algorithm to find the HCF of 105 and 120 (Ans.15)

2. Find the prime factorization of 234. (Ans. $2 \times 3^2 \times 13$)

3. Find the LCM and HCF of the pair of integers 13,11 (Ans. 1, 143)
4. Find the LCM and HCF of the pair of integers 510, 92 using fundamental theorem of arithmetic.

5. Find the missing number in the following factorization.

6. If the LCM (91, 26) = 182, then find the HCF (91, 26). (Ans. 13)

7. Find the LCM (96, 408) if the HCF is 24. (Ans. 1632)

8. Without actual division, state where the rational number $\frac{543}{225}$ is a terminating decimal expansion or non-terminating decimal expansion.

9. Show that $\frac{139}{2^35^3}$ will have terminating decimal expression.

10. State where $(\sqrt{6} + \sqrt{9})$ is rational or not.

11. Find LCM of 72, 80, and 120. (Ans. 720)

12. Prove that $\sqrt{11}$ is irrational.

13. The decimal expansion of a real number is 23.123456. If it is expressed as a rational number in the form of $\frac{p}{q}$, write the prime factors of q. (Ans: 2^65^6) (3 marks)

14. Find the largest number which divides 245 and 1029 leaving remainder 5 in each case. (Ans. 16)

15. Use Euclid’s division algorithm to find the HCF of 196, 38220. (Ans. 196)

LEVEL-2

16. LCM of two numbers is 2079 and HCF is 27. If one number is 297, find the other number. (Ans: 189)

17. Explain why $3 \times 11 \times 17 + 17 \times 7$ a composite number.

18. Find the LCM and HCF of the pair of integers 120, 70. Also verify that LCM x HCF = product of numbers.

19. After how many decimal places the decimal expansion of the rational number $\frac{11}{2^35^2}$ will terminate. (Ans 3)

20. Check whether $(15)^n$ can end with the digit 0. (2 marks)

21. Prove that $n^2 - n$ is divisible by 2 for every positive integer.

22. Show that any positive odd integer is of the form 4q+1 or 4q+3, where q is an integer.
23. What is the smallest number which when divided by 35, 56, 91, leaves the remainder 7 in each case. (Ans: 3647). (3 marks)

24. If \(d\) is the HCF of 45 and 27 find \(x\) and \(y\) satisfying \(d=27x+45y\). (Ans: \(x=2, y=-1\))

25. Using prime factorization method, find HCF and LCM of 72, 126, and 168. Also, show that \(\text{HCF} \times \text{LCM} = \text{product of the three numbers.}\)

26. In a school there are two sections A and B of class X. There are 48, 60 students in two sections respectively. Determine the least number of books required for the library so that the books can be distributed equally among the students. (Ans: 240)

27. Prove that \(5+7\sqrt{3}\) is an irrational number

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**LEVEL- 3**

(1 Mark)

28. For some integer \(m\), what is the form of every even integer. (Ans: \(2m\))

29. For some integer \(q\), what is the form of every odd integer \((Ans: 2q+1)\)

30. Find the least number that is divisible by all the numbers from 1 to 10 (both inclusive). (Ans: 2520)

31. If two positive integers \(p\) and \(q\) can be expressed as \(p = ab^2\) and \(q = a^3b;\) \(a, b\) being prime numbers, Find the LCM \((p, q)\). (Ans: \(a^3b^2\))

(3 marks)

32. Using Euclid's division algorithm, find which of the following pairs of numbers are co-prime: (i) 231, 396 (ii) 847, 2160

33. Show that the square of an odd positive integer is of the form \(8m + 1\), for some whole number \(m\).

34. Show that the square of any positive integer cannot be of the form \(5q + 2\) or \(5q + 3\) for any integer \(q = 2\)

35. If \(n\) is an odd integer, then show that \(n^2 - 1\) is divisible by 8.

36. Using Euclid's division algorithm, find the largest number that divides 1251, 9377 and 15628 leaving remainders 1, 2 and 3, respectively.

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**VALUE BASED QUESTIONS**

1. There are two old age homes in a city. 60 persons stay in one and 32 persons stay in other. What is the minimum number of sweets one should buy so that he is able to distribute them equally amongst the persons residing in any of the old age homes? What is the mathematical concept used? Which value is depicted?

2. Class X of a school has 120 boys and 114 girls. The principal of the school decides to have maximum number of mixed sections with equal number of boys and girls. What is the number of such sections? Mention the value depicted.

3. The teacher divided the class of 10 students in two groups of 5 each and told them to find the total number of prime factors in \((6^{11})(3^{12})(11^{7})\). What value the teacher wants to inculcate in students.

4. A Charitable trust donates 28 different books of Maths, 16 different books of Science and 12 different books of Social science to the poor students. Each student is given maximum number of books of one and only one subject of their interest and each student got equal number of books.
   a) Find the number of books each student got.
   b) Find the total number of students who got books.
   c) How does it help our society?
## ERROR ANALYSIS AND REMEDIATION

<table>
<thead>
<tr>
<th>Sl No</th>
<th>POSSIBLE ERRORS</th>
<th>REMEDIATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Students do not remove the brackets by multiplying all the terms in the bracket Ex 2x(k+3) is written as 2kx + 3</td>
<td>Use of BODMAS rule and distributive properties in operations should be practiced well.</td>
</tr>
<tr>
<td>2</td>
<td>Students cancel one of the terms of the numerator by a term/factor of a denominator</td>
<td>Students should be instructed and given practice on the proper method of cancellation of factors.</td>
</tr>
</tbody>
</table>
| 3     | In finding LCM and HCF of two or more numbers | → Stress on  
- Product of Smallest power of each common factor for LCM,  
- Product of greatest power of each common factor for HCF. |
| 4     | In identifying rational and irrational in the decimal form | → Stress and drill on more number of examples particularly-non-terminating repeating, Non-terminating and non-repeating. |
| 5     | Writing $a + \sqrt{b} = u\sqrt{b}$ | → Clarification using examples. |
| 6     | In verifying where the given rational number is terminating or non-terminating using prime factorization | drill on prime factorization of $2^a$ and $5^b$ for terminating rational numbers. |
| 7     | In finding decimal expansion of rational no. and square root of an irrational number | Peer-teaching and drilling. |
| 8     | In identifying decimal terminating decimal as a block and non-decimal terminating | Re-teaching the decimal expansion as a rational number using simple examples |
| 9     | In the formula $\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$, transposing the terms from LHS to RHS | Drill on linear equations |

## QUESTION BANK

1. Write whether every positive integer can be of the form $4q + 2$, where $q$ is an integer. Justify your answer.
2. “The product of two consecutive positive integers is divisible by 2”. Is this statement true or false? Give reasons.
3. “The product of three consecutive positive integers is divisible by 6”. Is this statement true or false”? Justify your answer.
4. Write whether the square of any positive integer can be of the form $3m + 2$, where $m$ is a natural number. Justify your answer.
5. A positive integer is of the form $3q + 1$, $q$ being a natural number. Can you write its square in any form other than $3m + 1$, i.e., $3m$ or $3m + 2$ for some integer $m$? Justify your answer.
6. The numbers 525 and 3000 are both divisible only by 3, 5, 15, 25 and 75. What is HCF (525, 3000)? Justify your answer.
7. Prove that $p + q$ is irrational, where $p$, $q$ are primes.
8. Can two numbers have 18 as their HCF and 380 as their LCM? Give reasons.
9. Without actually performing the long division, find if 987 10500 will have terminating or non-terminating (repeating) decimal expansion. Give reasons for your answer.
10. A rational number in its decimal expansion is 327.7081. What can you say about the prime factors of $q$, when this number is expressed in the form $\frac{p}{q}$? Give reasons.
11. Show that the cube of a positive integer of the form $6q + r$, $q$ is an integer and $r = 0, 1, 2, 3, 4, 5$ is also of the form $6m + r$. 

12. Prove that one and only one out of \( n, n + 2 \) and \( n + 4 \) is divisible by 3, where \( n \) is any positive integer.
13. Prove that one of any three consecutive positive integers must be divisible by 3. For any positive integer \( n \), prove that \( n^3 - n \) is divisible by 6.
14. Show that one and only one out of \( n, n + 4, n + 8, n + 12 \) and \( n + 16 \) is divisible by 5, where \( n \) is any positive integer. [Hint: Any positive integer can be written in the form \( 5q, 5q+1, 5q+2, 5q +3, 5q+4 \)]

**ACTIVITIES**

1. Cross word puzzle
2. Quiz on real numbers.
3. Home assignment-short and long answer question.
4. Representing irrational numbers on the number line and its magnification-spiral method.
5. Remedial work sheets on finding decimal representation of rational numbers.
6. Seminar on Ancient Indian mathematicians and their contributions on Real numbers.

**PROJECTS**

1. Pictorial representation of number system.
2. Math Vs Computer: Make a flow chart to elaborate and explain
3. EUCLID’s ALGORITHM.
4. Take a help of a computer science seniors/friends/teacher to convert your flow chart into a computer to ease your calculation work.
5. Math journal.
6. Evolution of Number system-journey from counting numbers to real numbers.
7. Extension of real number system-knowledge of complex numbers.
8. Application of real number system in day to day life.

**POWER POINT PRESENTATIONS**

**WEB LINKS**

- Irrational Numbers- [http://www.wtamu.edu/academic/anns/mps/math/mathlab/int_algebra/index.htm](http://www.wtamu.edu/academic/anns/mps/math/mathlab/int_algebra/index.htm)
- Teaching Irrational Numbers- [https://www.youtube.com/watch?v=qkxLNSwop7c](https://www.youtube.com/watch?v=qkxLNSwop7c)
CHAPTER-2
POLYNOMIALS

INTRODUCTION
This chapter deals about the geometrical representation of linear and quadratic polynomials and geometrical meaning of their zeroes. The relationship between the zeroes and the coefficients of a polynomial is also discussed.

EXPECTED LEARNING OUTCOMES
1. To recall and review polynomial, degree, coefficients, constants, zeroes, factors
2. To identify linear, quadratic and cubic polynomial.
3. To remember general form of a quadratic polynomial in \( x \) as \( ax^2 + bx + c \), \( a,b,c \in R \) and \( a \neq 0 \).
4. To analyze that the zeroes of a polynomial are the x-coordinate of the point where the graph of \( y = p(x) \) intersects the x-axis.
5. To interpret and understand that a quadratic polynomial can have at most two zeroes and a cubic polynomial can have three zeroes.
6. To review and recall splitting middle term of a quadratic polynomial.
7. To remember and interpret that if \( \alpha, \beta \) are the roots of the quadratic polynomial \( ax^2 + bx + c \), then \( \alpha + \beta = \frac{-b}{a} \), \( \alpha \beta = \frac{c}{a} \)
8. To Remember and interpret that if \( \alpha, \beta, \gamma \) are the roots of the cubic polynomial then \( \alpha + \beta + \gamma = \frac{-b}{a} \), \( \alpha \beta + \beta \gamma + \gamma \alpha = \frac{c}{a} \), \( \alpha \beta \gamma = \frac{d}{a} \)
9. To Solve problems based on the relation between zeroes and the coefficient of the polynomial.
10. To Understand that given any polynomial \( p(x) \) and any non-zero polynomial \( g(x) \), there exists \( q(x), r(x) \) such that \( p(x) = q(x)q(x) + r(x) \), where \( r(x) = 0 \) or degree \( r(x) < \) degree \( g(x) \).
11. To Find the remaining zeroes of a bi quadratic polynomial if two of its zeroes are given.
Polynomial of degree 1, 2, 3 are called linear, quadratic, cubic.

- Geometrical meaning of the zeroes of a polynomial
- Relationship between zeroes and coefficients of a polynomial
- If the value of a polynomial at k is 0, k is a zero of the polynomial
- A polynomial of degree n can have at most n real zeroes
- Division for polynomial
GRADED EXERCISES

LEVEL – 1

1. If \( p(x) = 2x^2 - 3x + 5 \), then find \( p(-1) \). (ans :10)
2. Find the zero of the polynomial \( 3x + 2 \).
3. The following figure shows the graphs of \( y = p(x) \), where \( p(x) \) is a polynomial.

Find the zeroes of the polynomial.

![Graph of a polynomial](image)

4. Find the zeroes of the polynomial \( 2x^2 - 25 \)  
   (Ans. \( \pm \frac{5}{\sqrt{2}} \))
5. Find the quadratic polynomial whose roots are \( 3 + \sqrt{5} \) and \( 3 - \sqrt{5} \) (Ans. \( x^2 - 6x + 7 \))
6. If one zero of \( 2x^2 - 3x + k \) is reciprocal to the other, then find the value of \( k \) (ans: \( k= 2 \))
7. Find a quadratic polynomial whose sum and product of roots are \( 2, \frac{-3}{5} \).
   (Ans : \( 5x^2 - 10x - 3 \))
8. If \( \alpha, \beta \) are the zeroes of the quadratic polynomial \( x^2 - 6x + a \), find the value of \( a \)
   If \( 3\alpha + 2\beta = 20 \). (Ans: -16)
9. Divide the polynomial \( x^3 - 3x^2 + 5x - 3 \) by \( x^2 - 2 \).
10. Is \( x+2 \) a factor of \( 2x^2 + 3x + 1 \).

LEVEL – 2

11. If \( \alpha, \frac{1}{\alpha} \) are the zeroes of the polynomial \( 4 - 2x + (k - 4) \), find the value of \( k \). (Ans: \( k=8 \))
12. The graph of \( y = f(x) \), where \( f(x) \) is a polynomial in \( x \) is given below. Find the number of zeroes lying between \(-2\) to \(0\).

![Graph of a polynomial](image)

13. Show that \( 1, -1 \) and \( 3 \) are the zeroes of the polynomial \( x^3 - 3x^2 - x + 3 \).
14. If \( (x + a) \) is a factor of \( 2x^2 + 2ax + 5x + 10 \), find \( a \). (ans: \( a=2 \))
15. If the polynomial \( 6x^4 + 8x^3 + 17x^2 + 21 + 7i \) is divided by the polynomial \( 13x^2 + 4x + 1 \) and the remainder is \( ax + b \), find the value of \( a, b \). (Ans: \( a=1, b=2 \))
16. If \( \alpha \) and \( \beta \) are the zeroes of the quadratic polynomial \( f(x) = x^2 - 5x + 6 \) then find the value of \( \frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta \). (Ans : -31)
LEVEL-3

17. Find all the zeroes of the polynomial $2x^4 - 3x^3 - 3x^2 + 6x - 2$, if two of its zeroes are $\sqrt{2}, -\sqrt{2}$. (Ans. $1, 1$)

18. What must be added to $8x^4 + 14x^3 - 2x^2 + 7x - 8$ so that the resulting polynomial is exactly divisible by $4x^2 + 3x - 2$. (Ans. 10 - 14x)

19. Find the values of $a, b$ so that $x^4 + x^3 + 8x^2 + ax + b$ is divisible by $x^2 + 1$. (Ans. $a=1, b=7$)

20. Divide $2x^4 - 9x^3 + 5x^2 + 3x - 8$ by $1 - 4x + x^2$ and verify division algorithm.

21. Find the zeroes of $3\sqrt{2}x^2 + 13x + 6\sqrt{2}$ and verify the relation between the zeroes and coefficients of the polynomial.

22. If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are $a-b, a, a+b$, find $a, b$ ($a=1, b=\pm\sqrt{2}$)

23. Verify where $\frac{1}{2}, 1, -2$ are the zeroes of the polynomial $2x^3 + x^2 - 5x + 2$. Also verify the relationships between the zeroes and the coefficients.

VALUE BASED QUESTIONS

1. Defense Minister decided to build a war memorial in the memory of KARGIL MARTYRS. Further he decided that it should be in the form of a Uniform curve with one smooth vertex at the top as shown in the adjoining figure. What will be the degree of the polynomial which will describe the outer border of the memorial? Mention the value that the minister depicts by building the war memorial.

2. A rocket is launched by the army to bust the hide outs of the militants. The path of the rocket is as shown in the adjoining figure. If the equation of the path represents a polynomial, what is the number of zeroes of the polynomial? What value the army represent?

3. On the occasion of Diwali Rashmi wants to donate blankets to poor people. She purchased two blankets more than the number of poor ones. If the cost of each blanket is Rs. 2 less than the number of poor ones. Find the amount she paid for blankets. What values are reflected from the question?

4. Product of timings of 3 major activities for one student out of 24 hrs is $x^3 + 12x^2 + 44x + 48$ and product of two of its activities is $x^2 + 6x + 8$. If quotient of these two is the time of study. Find the number of hours for a student to study. What values reflect, if least is given to activity playing.

5. Naveen went to school by cycle at the speed of $x$ km/hr and covers the distance in $(x-2)$ hrs. His brother goes to college by scooter covering a distance $y$ km in $(y-4)$ hrs. Represent the distance covered by both in the form of a polynomial. What are the values reflected here to improve the HEALTH and ENVIRONMENT.
## QUESTION BANK

**Answer the following and justify:**

1. Can \( x^2 - 1 \) be the quotient on division of \( x^6 + 2x^3 + x - 1 \) by a polynomial in \( x \) of degree 5?
2. What will the quotient and remainder be on division of \( x^2 + bx + c \) by \( px^3 + qx^2 + rx + s, p \neq 0 \)?
3. If on division of a polynomial \( p(x) \) by a polynomial \( g(x) \), the quotient is zero, what is the relation between the degrees of \( p(x) \) and \( g(x) \)?
4. Given that the zeroes of the cubic polynomial \( x^3 - 6x^2 + 3x + 10 \) are of the form \( a, a + b, a + 2b \) for some real numbers \( a \) and \( b \), find the values of \( a \) and \( b \) as well as the zeroes of the given polynomial.
5. Find \( k \) so that \( x^2 + 2x + k \) is a factor of \( 2x^4 + x^3 - 14x^2 + 5x + 6 \). Also find all the zeroes of the two polynomials.
6. Given that \( x - 5 \) is a factor of the cubic polynomial \( x^3 - 3\sqrt{5}x^2 + 13x - 35 \), find all the zeroes of the polynomial.
7. For which values of \( a \) and \( b \), are the zeroes of \( q(x) = x^3 + 2x^2 + a \) also the zeroes of the polynomial \( p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b \)? Which zeroes of \( p(x) \) are not the zeroes of \( q(x) \)?
ACTIVITIES

1. Using the given constants and variables polynomials forming polynomials.
2. Finding zeroes of a polynomial using graph.
3. Verifying the relations between zeroes and the coefficient of a polynomial of degree two and three.
5. Home assignment-short and long answer question.
6. Quiz.
7. Remedial work sheets.
8. Seminar on mathematicians and their contributions on polynomials.

PROJECTS

1. Verifying the relations between zeroes and the coefficient of a polynomial of degree 4.
2. Drawing graph of quadratic polynomial and finding its zeroes. Finding the relation between the sign of coefficient of $x^2$ and the shape of the graph of the quadratic polynomial.

POWER POINT PRESENTATION

WEBLINKS

Introduction to polynomials - https://www.youtube.com/watch?v=jWkbSxqDgQI
Zeros of a polynomials - https://www.youtube.com/watch?v=QXOLozXkAvs
Cubic polynomials - https://www.youtube.com/watch?v=tdeQwInVcuY
Geometrical meaning of zeros of a polynomials - https://www.youtube.com/watch?v=W3dP527oT7A
CHAPTER-3

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

INTRODUCTION

In this chapter the notion of system of simultaneous linear equations is introduced as a pair of linear equations in two variables. Also the study about a pair of linear equations in two variables, its algebraic methods of solutions as well as graphical method is discussed. In the end the applications of linear equations in two variables in solving simple problems from different areas are given.

EXPECTED LEARNING OUTCOMES

1. To identify and to understand the general form of a pair of linear equations in two variables.
2. To represent and to solve a pair of linear equations in two variables graphically and algebraically.
3. To differentiate from the graph of a system of linear equations as consistent or inconsistent.
4. To solve a pair of linear equations in two variables using substitution method, elimination method and cross-multiplication method.
5. To understand the condition for consistency and inconsistency of a pair of linear equations in two variables.
6. To interpret and to solve the word problems to a pair of linear equations in two variables.
7. To identify and to solve the equations reducible to a pair of linear equations in two variables.
GRADED EXERCISES

LEVEL-1

1. Show that the pair of equations $2x-3y=6$, $x+y=1$. has a unique solution.
2. Show that the pair of equations $2y=4x-6$, $2x=y+3$, has infinite solutions.
3. For what value of $k$, $2x+3y=4$, $(k+2)x+6y=3k+2$ will have infinitely many solutions.  
   (Ans. $k=2$)
4. Is the pair of equations: $x+2y=4=0$,
   $2x+4y-2=0$ consistent.
5. Solve algebraically the pair of equations:
   $2x-y=5$,
   $3x+2y=11$. (Ans. $x=3$, $y=1$)
6. Solve by the method of cross multiplication the pair of equations:
   $2x+3y+8=0$
7. Solve graphically the pair of equations:
   \[2x + y = 6\]
   \[2x - y + 2 = 0.\]

8. Solve the following system of equations graphically and from the graph, find the points where
   these lines intersect the y-axis:
   \[x - 2y = 2,\]
   \[3x + 5y = 17.\]

9. Solve the following system of equations graphically and find the vertices of the triangle formed
   by these lines and the x-axis:
   \[4x - 3y + 4 = 0,\]
   \[4x + 3y - 20 = 0.\]

10. Solve graphically:
    \[x - y = 1\]
    \[2x + y = 8.\]
    Shade the region bounded by these lines and y-axis. Also find its area.

11. Draw the graph of the equations \(x = 5,\) \(y = -4.\) Also find area of rectangle so formed by
    these lines x-axis and y-axis.

**LEVEL-2**

12. If \(2x-3y=7\) and \((a+b)x - (a+b-3)y=4a+b\) have infinite solution, find \(a,\ b\)  \(\text{Ans. } (-5,1)\)

13. Solve: \[\frac{x}{a} + \frac{y}{b} = 2\]
    \[ax-by = a^2 - b^2.\]
    \(\text{Ans: } x=a, \ y=b\)

14. Solve: \[23x+29y=98\]
    \[29x+23y=110.\]
    \(\text{Ans. } x=3, y=1).\)

15. Solve: \[152x - 378y = -74\]
    \[-378x+152y=-604\]

16. The sum of the digits of a two digit number is 12. The number obtained by interchanging
    the two digits exceeds the given number by 18. Find the number.
    \(\text{Ans: 57}\)

17. The monthly incomes of \(A\) and \(B\) are in the ratio of \(5 : 4\) and their monthly expenditures
    are in the ratio of \(7 : 5.\) If each saves Rs. 3000 per month, find the monthly income of each.
    \(\text{Ans: Rs.10000, Rs.8000}\)

18. A part of monthly hostel charges is fixed and the remaining depends on the number of days one
    has taken food in the mess. When a student \(A\) takes food for 20 days, she has to pay
    Rs. 1000 as hostel charges whereas a student \(B,\) who takes food 26days, pays Rs 1180 as hostel
    charges. Find the fixed charges and the cost of the food per day.
    \(\text{Ans: x=Rs.400, y=Rs 30.}\)

19. Father’s age is 3 times the sum of ages of his two children. After 5 years his age will be twice the
    sum of ages of the two children. Find the age of father
    \(\text{Ans: 45}\)

20. Two numbers are in the ratio \(5 : 6.\) If 8 is subtracted from each of the numbers, the ratio becomes
    \(4 : 5.\) Find the numbers.
    \(\text{Ans: 40,48}\)
21. If 4 times the area of a smaller square is subtracted from the area of a larger square, the result is 144 m². The sum of the areas of the two squares is 464 m². Determine the sides of the two squares
   (Ans: Side=8m)

LEVEL-3

22. Solve:
   \[
   \frac{x+1}{2} + \frac{y-1}{3} = 8
   \]
   \[
   \frac{x-1}{3} + \frac{y+1}{2} = 9.
   \]
   (Ans: x=7, y=13)

23. Solve:
   \[
   \frac{2}{x-1} + \frac{3}{y+1} = 2
   \]

24. Solve:
   \[
   \frac{13}{6} = \frac{3}{x-1} + \frac{2}{y+1}
   \]
   (Ans: x=3, y=2)

25. Solve:
   \[
   \frac{7x-2y}{xy} = 5
   \]
   (Ans: x=1, y=1)

26. In the figure, ABCD is a rectangle. Find the values of x and y.

27. In the figure, ABCD is a parallelogram. Find the values of x and y.

28. Nine times a two-digit number is the same as twice the number obtained by interchanging the digits of the number. If one digit of the number exceeds the other number by 7, find the number.
   (Ans: number=18)

29. A man travels 370 km partly by train and partly by car. If he covers 250 km by train and rest by car, it takes him 4 hours. But if he travels 130 km by train and rest by car, he takes 18 minutes longer. Find the speed of the train and that of the car.
   (Ans: speed of the train=100km/hr, speed of the car =80km/hr)
A boat goes 12 km upstream and 40 km downstream in 8 hours. It goes 16 km upstream and 32 km downstream in the same time. Find the speed of the boat in still water and the speed of the stream.

(Ans: 6km/hr, 2km/hr)

VALUE BASED QUESTIONS

1. A boat goes 30 km upstream and 44 km downstream in 10 hrs. In 13 hours it can go 40 km upstream and 55 km downstream. Determine the speed of the stream and that of the boat. What value to be learnt in real life from this problem?

2. A class of 20 boys and 15 girls is divided into groups so that each group has x boys and y girls. Find x, y and what values are referred in a class.

3. 2 men and 5 women can together finish a piece of work in 4 days, while 3 men and 6 women can finish it in 3 days. Find the time taken by 1 man alone to finish the work, and also that taken by 1 woman alone. Write one benefit of gender equality.

4. A person invested some amount @ 12% simple interest and some other amount @ 10% simple interest. He received a yearly interest of Rs. 13000. But if he had interchanged the invested amounts, he would have received Rs. 400 more as interest. How much amount did he invest at different rates? Mention any one advantage of savings for a country.

5. It takes 12 hours to fill a swimming pool using two pipes. If the pipe of larger diameter is used for 4 hours and the pipe of smaller diameter for 9 hours, only half the pool can be filled. How long would it take for each pipe to fill the pool separately? Suggest one method to save water.

ERROR ANALYSIS AND REMEDIATION

<table>
<thead>
<tr>
<th>ERROR</th>
<th>REMEDIATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Drawing axes as line segment.</td>
<td>Stress on arrow marks for axes(lines)</td>
</tr>
<tr>
<td>2. Not naming the x-axis and y-axis.</td>
<td>Stress on XOY and YOY1</td>
</tr>
<tr>
<td>3. Writing $x-2y-3=0$ as $y=\frac{x-3}{2}$</td>
<td>Drill on transposing of the terms</td>
</tr>
<tr>
<td>4. Confusion over shading of triangle formed by the lines and any one of the axes.</td>
<td>Drill on drawing graph of lines and the proper area.</td>
</tr>
<tr>
<td>5. Writing $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ infinitely many solutions.</td>
<td>Correcting as no solution.</td>
</tr>
<tr>
<td>6. In the method of cross multiplication, writing $x \ y \ 1 \ b_1 \ c_1a_1 \ b_1 \ b_2 \ c_2a_2 \ b_2$</td>
<td>Teaching the correct diagram.</td>
</tr>
<tr>
<td>7. Solve $\frac{2}{x} + \frac{3}{y} = 13$</td>
<td>$\frac{5}{x} - \frac{4}{y} = -2$. Writing the equations as</td>
</tr>
<tr>
<td>8. In the problems of upstream and downstream writing speed in the upstream as x+3km/hr and downstream as x-3km/hr.</td>
<td>x and y are reciprocals of a and b.</td>
</tr>
</tbody>
</table>

Re-teaching of finding the speed.
1. Two straight paths are represented by the equations $x - 3y = 2$ and $-2x + 6y = 5$. Check whether the paths cross each other or not.

2. Write a pair of linear equations which has the unique solution $x = -1$, $y = 3$. How many such pairs can you write?

3. Write an equation of a line passing through the point representing solution of the pair of linear equations $x + y = 2$ and $2x - y = 1$. How many such lines can we find?

4. For which values of $p$ and $q$, will the following pair of linear equations have infinitely many solutions?

   
   \[4x + 5y = 2 (2p + 7q) \quad x + (p + 8q) \quad y = 2q - p + 1\]

5. There are some students in the two examination halls A and B. To make the number of students equal in each hall, 10 students are sent from A to B. But if 20 students are sent from B to A, the number of students in A becomes double the number of students in B. Find the number of students in the two halls.

6. In a competitive examination, one mark is awarded for each correct answer while 1 2 mark is deducted for every wrong answer. Jayanti answered 120 questions and got 90 marks. How many questions did she answer correctly?

7. Jamila sold a table and a chair for Rs 1050, thereby making a profit of 10% on the table and 25% on the chair. If she had taken a profit of 25% on the table and 10% on the chair she would have got Rs 1065. Find the cost price of each.

8. A person, rowing at the rate of 5 km/h in still water, takes thrice as much time in going 40 km upstream as in going 40 km downstream. Find the speed of the stream.

9. Draw the graphs of the equations $x = 3$, $x = 5$ and $2x - y - 4 = 0$. Also find the area of the quadrilateral formed by the lines and the $x$–axis.

10. A shopkeeper sells a saree at 8% profit and a sweater at 10% discount, thereby, getting a sum Rs 1008. If she had sold the saree at 10% profit and the sweater at 8% discount, she would have got Rs 1028. Find the cost price of the saree and the list price (price before discount) of the sweater.

11. Susan invested certain amount of money in two schemes A and B, which offer interest at the rate of 8% per annum and 9% per annum, respectively. She received Rs 1860 as annual interest. However, had she interchanged the amount of investments in the two schemes, she would have received Rs 20 more as annual interest. How much money did she invest in each scheme?

12. Vijay had some bananas, and he divided them into two lots A and B. He sold the first lot at the rate of Rs 2 for 3 bananas and the second lot at the rate of Re 1 per banana, and got a total of Rs 400. If he had sold the first lot at the rate of Re 1 per banana, and the second lot at the rate of Rs 4 for 5 bananas, his total collection would have been Rs 460. Find the total number of bananas he had.
ACTIVITIES/PROJECTS

1. Formation of linear equations in two variables.
2. Formation of pair of linear equations in two variables
3. Solution of linear equations in two variables using Graph.
4. Comparing the ratios of coefficients of pair of linear equations in two variables and verifying whether the system is consistent or inconsistent.
5. Home assignment-short and long answer question.
6. Remedial work sheets.
7. Seminar on Math and its correlation with other subjects.
   Project
8. Understanding and obtaining the system of three linear equation in two Variables and drawing their graphs. Identifying the area bounded by the lines. Finding the area of the region using different methods.

POWER POINT PRESENTATIONS

Pair of linear equations:
http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut49_systwo.htm
CHAPTER 6

TRIANGLES

INTRODUCTION

- Two congruent figures are similar. But, similar figures need not be congruent.
- Circles having different radii are similar.
- Shadow of an object is similar to the object itself. But the shadow and the corresponding object may not be congruent.
- \( \Delta ABC \cong \Delta DEF \) but \( \Delta ABC \) may not be congruent to \( \Delta EDF \) for the same respective triangles. Hence similarity can be introduced.
- For an equilateral triangle 6 congruencies are possible. It can show 6 similarities also. But it is not possible for other triangles.

EXPECTED LEARNING OUTCOMES

1. To identify two similar triangles.
2. To understand the Basic Proportionality Theorem & its converse
3. To identify the Criteria of Similarity of triangles
   (a) AA or AAA similarity criterion.
   (b) SAS similarity criterion
   (c) SSS similarity criterion.
4. To prove Areas of Similar triangles are equal.
5. To identify the right triangle and its properties
6. To prove Pythagoras theorem & its Converse
7. To solve sums by using the above said theorems.
GRADED EXERCISES

LEVEL 1

1. In fig.1, DE // BC, if BD = 3 cm, AD = 2 cm, AE = 4 cm, then find the value of EC.

2. If \( \Delta ABC \) and \( \Delta DEF \) are two similar triangles such that \( \angle A = 45^\circ \) and \( \angle F = 57^\circ \), then find \( \angle C \).

3. The length of the diagonals of a rhombus are 24 cm and 32 cm. Then find the length of the side of the rhombus.

4. In \( \Delta ABC \), DE is a line such that D and E are points on AB and CA and \( \angle B = \angle AED \). Show that \( \Delta ABC \sim \Delta AED \).

5. In \( \Delta ABC \), a line PQ parallel to BC intersect AB at P and AC at Q. If AP:PB = 1:2, the find the area(\( \Delta APQ \)) : area(\( \Delta ABC \)).

6. P and Q are points on sides AB and AC respectively, of \( \Delta ABC \). If AP = 3 cm, PB = 6 cm, AQ = 5 cm and QC = 10 cm, show that BC = 3 PQ.

7. It is given that \( \Delta ABC \sim \Delta EDF \) such that AB = 5 cm, AC = 7 cm, DF = 15 cm and DE = 12 cm. Find the lengths of the remaining sides of the triangle. BC = 6.25 cm, EF = 16.8 cm.

8. Lines AC and BD intersect at P, AB and CD are joined such that \( \angle A = \angle C \), AB = 6 cm, BP = 15 cm, AP = 12 cm and CP = 4 cm, then find the lengths of PD and CD.

9. Find the altitude of an equilateral triangle of side 8 cm.

Corresponding sides of two similar triangles are in the ratio of 2 : 3. If the area of the smaller triangle is 48 cm\(^2\), find the area of the larger triangle.

LEVEL 2

10. O is any point inside a rectangle ABCD. Prove that \( OB^2 + OD^2 = OA^2 + OC^2 \).

11. AD is an altitude of an equilateral triangle ABC. On AD as base, another equilateral triangle ADE is constructed. Prove that area(\( \Delta ADE \)) : area(\( \Delta ABC \)) = 3 : 4.

12. ABC is a right triangle right angled at C. Let BC = a, CA = b, AB = c and let p be the length of perpendicular from C on AB. Prove that cp = ab.

13. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of another triangle PQR. Show that \( \Delta ABC \sim \Delta PQR \).

14. In a trapezium ABCD, O is the point of intersection of AC and BD, AB parallel to CD and AB = 2 CD. If the area of \( \Delta AOB = 84 \) cm\(^2\), then find area of that \( \Delta COD \). (Ans. 21 cm\(^2\))

15. In figure 1, DE parallel to OQ and DF parallel to OR, Show that EF parallel to QR.

Figure 1
16. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that \( \triangle ABE \sim \triangle CFB \). (3 mark)

17. In \( \triangle ABC \), DE parallel to BC, where D and E are on the side AB and AC respectively, if \( \text{AD} : \text{DB} = 5:4 \), find \( \frac{\text{Area}(\triangle DEF)}{\text{Area}(\triangle ABC)} \). (3 mark)

18. Prove that the area of the semicircle drawn on the hypotenuse of a right angled triangle is equal to the sum of the areas of the semicircles drawn on the other two sides of the triangle. (3 mark)

19. O is the point of intersection of the diagonals AC and BD of a trapezium ABCD with AB parallel to DC. Through O, a line segment PQ is drawn parallel to AB meeting AD in P and BC in Q. Prove that PO = QO. (3 mark)

**LEVEL 3**

20. The diagonals of a quadrilateral ABCD intersect each other at the point O such that \( \frac{AO}{BO} = \frac{CO}{DO} \). Show that ABCD is a trapezium. (4 mark)

21. In the given figure PA, QB and RC are each perpendicular to AC. If \( \text{AP} = x \), \( \text{QB} = z \), \( \text{RC} = y \) and \( \text{BC} = b \), then prove that \( \frac{1}{x} + \frac{1}{y} = \frac{1}{z} \). (4 mark)

22. In an equilateral triangle ABC, D is a point on the side BC, such that BD = \( \frac{1}{3} \) BC. Prove that \( 9AD^2 = 7AB^2 \). (4 mark)

23. P and Q are the mid points of side CA and CB respectively of \( \triangle ABC \) right angled at C. Prove that \( 4(AQ^2 + BP^2) = 5AB^2 \). (4 mark)

24. In a right \( \triangle ABC \), right angled at C, P and Q are points of the sides CA and CB respectively, which divide these sides in the ratio 2:1. Prove that
   (a) \( 9AQ^2 = 9AC^2 + 4BC^2 \)
   (b) \( 9BP^2 = 9BC^2 + 4AC^2 \)
   (c) \( 9(AQ^2 + BP^2) = 13AB^2 \) (4 mark)

25. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals. (4 mark)

26. In a quadrilateral ABCD, \( \angle A + \angle D = 90^\circ \). Prove that \( AC^2 + BD^2 = AD^2 + BC^2 \). (4 mark)

27. In \( \triangle ABC \), D and E trisects BC. Prove that \( 8AE^2 = 3AC^2 + 5AD^2 \). (4 mark)

28. In \( \triangle ABC \), P is the mid - point of BC and Q is the mid - point of AP. If BQ when produced meets AC at R, prove that \( \frac{RA}{CA} = \frac{1}{3} \). (4 mark)

29. If a line is drawn parallel to one side of the triangle intersecting the other two sides at distinct points, then the other two sides are divided with the same ratio. Using the above prove the following: In the figure given DE//AC and DC//AP, prove that \( \frac{BE}{EC} = \frac{BC}{CP} \). (4 mark)
VALUE BASED QUESTIONS

1. A ‘No Smoking’ campaign was organized by certain teenagers in a society for which some posters were prepared. Each poster was made on cardboard in the shape of an equilateral triangle (say ABC) and a perpendicular (say AD) was drawn from one vertex to the opposite side (BC here). Show that $3 \ AB^2 = 4 \ AD^2$.
   What values do these teenagers possess?

2. Ravi prepared two posters ‘NATIONAL INTEGRATION’ for decoration of Independence Day on triangular sheets (say ABC and DEF). The sides AB and AC and the perimeter $P_1$ of $\Delta$ABC are respectively three times the corresponding sides DE and DF and the perimeter $P_2$ of $\Delta$DEF. Are the two triangular sheets similar? If yes find $\frac{ar(\Delta ABC)}{ar(\Delta DEF)}$. What values can be inculcated through celebration of national festivals?

3. A farmer has filed in the form of triangle. He wants to give this land to his 4 sons equally. Explain how will he separate his land? Also write the values given by the query.

4. A mathematics teacher shows circles of different radii to 35 students of a class and asked what do you observe? What have you learnt from it?

5. On occasion of Independence Day three persons buy National flags costing Rs.5, Rs.50 and Rs.500 of different size and of different material. What would you observe in these flags? What value is depicted by purchasing flags on independence day.

ERROR ANALYSIS

<table>
<thead>
<tr>
<th>Sl no</th>
<th>POSSIBLE ERRORS</th>
<th>REMEDIATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Some students taking similar figures are congruent in areas.</td>
<td>Congruent figures have equal areas but not for similar figures, eg: Two CDs have equal area but two circular bindies of different size are similar but have different areas.</td>
</tr>
<tr>
<td>2</td>
<td>While taking proportional parts of similar figures are in the ratio for $\Delta ABC \sim \Delta DEF$, just taking $\frac{AB}{DE} = \frac{BC}{AC} = \frac{EF}{DF}$.</td>
<td>One to one corresponding sides should consider.</td>
</tr>
<tr>
<td>3</td>
<td>Proving as a right triangle while sides are given as $a = 5$ cm, $b = 4$ cm and $c = 3$ cm, students take for right angled triangle as $a^2 + b^2 = c^2$.</td>
<td>It has to be emphasized that $b^2 + c^2 = 16 + 9 = 25 = c^2$.</td>
</tr>
<tr>
<td>4</td>
<td>When a general question like in how many ways two equilateral triangle were congruent or similar students, the answer was 1.</td>
<td>But it is possible 6 ways for two equilateral triangle were congruent or similar.</td>
</tr>
<tr>
<td>5</td>
<td>Students do not first recognize the right angle and apply Pythagoras theorem as $AB^2 + BC^2 = AC^2$.</td>
<td>Instruct the students to identify right angle before applying the Pythagoras theorem.</td>
</tr>
</tbody>
</table>
QUESTION BANK

1. Prove that three times the sum of the squares of the sides of a triangle is equal to four times the sum of the squares of the medians of the triangle.

2. If \( A \) be the area of a right triangle and \( b \) one of the sides containing the right angle, prove that the length of the altitude on the hypotenuse is \( \sqrt{\frac{2Ab}{b^2 + 4A^2}} \).

3. If \( ABC \) is an obtuse angled triangle, obtuse angled at \( B \) and if \( AD \perp CB \)
   Prove that \( AC^2 = AB^2 + BC^2 + 2BC \cdot BD \)

4. In \( \triangle ABC \), \( P \) is any point in its interior such that \( AP^2 = PB^2 + PC^2 \). Show that \( \angle BPC = 150^\circ \).

5. \( ABCD \) is a parallelogram in the given figure, \( AB \) is divided at \( P \) and \( CD \) and \( Q \) so that \( AP:PB=3:2 \) and \( CQ:QD=4:1 \). If \( PQ \) meets \( AC \) at \( R \), prove that \( AR=AC \).

6. Is the triangle with sides 25 cm, 5 cm and 24 cm a right triangle? Give reasons for your answer.

7. In \( \triangle ABC \), \( D \) and \( E \) are points such that the line \( BC \) divides the triangle \( \angle D = \angle C \). Then is it true that \( \triangle ADE \sim \triangle ACB \). Why?

8. \( D \) is a point on side \( QR \) of \( \triangle PQR \) such that \( PD \) is perpendicular to \( QR \). Will it be correct to say that \( \triangle PQD \sim \triangle RPQ \)? Why?

9. In \( \triangle PQR \), \( PD \) perpendicular to \( QR \) such that \( D \) lies on \( QR \). If \( PQ = a, PR = b, QD = c \) and \( DR = d \). Prove that \( (a + b)(a - b) = (c + d)(c - d) \).

10. For going to a city \( B \) from city \( A \), there is a route via city \( C \) such that \( AC \) Perpendicular to \( CB \), \( Ac = 2x \) km and \( CB = 2(x + 7) \) km. It is proposed to construct a 26 km highway which directly connects the two cities \( A \) and \( B \). Find how much distance will be saved in reaching city \( B \) from city \( A \) after the construction of the highway.

ACTIVITIES/PROJECTS

1. To verify Basic Proportionality Theorem using parallel lines board, triangle cutouts.

2. To verify Pythagoras theorem by performing an activity. (The area of the square constructed on the hypotenuse of a right angled triangle is equal to the sum of the areas of squares constructed on the other two sides of a right angled triangle.

3. To verify “The ratio of the areas of two similar triangles is equal to the ratio of the square of their corresponding sides” by performing an activity.

4. To verify the formula for the area of a triangle by graphical method.
   Arrange the triangle placed on the graph sheet – count the squares of the graph sheet occupied by the triangle.

5. Downloading from website various method of proving Pythagoras theorem
POWER POINT PRESENTATIONS

WEB LINKS

- Theorems about Similar Triangles Web site
  - http://www.mathsisfun.com/geometry/triangles-similar.html
- Similar triangle Theorem web site:
  - SAS Similarity Theorem- Wiki Sucher
- Similar Triangles (with worked solutions & videos) Web site and is
  - http://www.onlinemathlearning.com/similar-triangles.html
- Similar polygon and triangles
  - http://www.mathcaptain.com/geometry/similar-triangles.html#
  - http://www.helpingwithmath.com/by_subject/geometry/geo_similar_triangles_8g4.htm
- Similar triangles and solving simple sums Exercise question
  - https://images.search.yahoo.com/yhs/search;_ylt=A86.J3at0TdVFh0ArWYnnIIQ;_ylu=X3oDMTB0MzkwOG5yBHNIYwNzYwRjb2xvA2dx
- Pythagorean theorem
  - https://www.youtube.com/watch?v=_A6bM8wOCysExplanation about Pythagorean theorem
  - https://www.khanacademy.org/math/basic-geo/basic-geo-pythagorean-topic/basic-geo-pythagorean-proofs/v/bhaskara-s-proof-of-pythagorean-theorem-avi
CHAPTER-8
INTRODUCTION TO TRIGONOMETRY

INTRODUCTION

- As you see, the word itself refers to three angles - a reference to triangles.
- Trigonometry is primarily a branch of mathematics that deals with triangles, mostly right triangles.
- In particular the ratios are relationships between the triangle's sides and angles.

EXPECTED LEARNING OUTCOMES

1. To know about Trigonometric Ratios of a right angled triangle Sin $\theta$, Cos $\theta$, tan $\theta$, Cosec $\theta$, Sec $\theta$ and Cot $\theta$.
2. To identify the relationship between different trigonometric ratios.
3. To understand the common trigonometric identities:
   a) $\sin^2 \theta + \cos^2 \theta = 1$ for $0^\circ \leq \theta \leq 90^\circ$
   b) $1 + \tan^2 \theta = \sec^2 \theta$ for $0^\circ \leq \theta \leq 90^\circ$
   c) $1 + \cot^2 \theta = \cosec^2 \theta$ for $0^\circ < \theta \leq 90^\circ$
4. To understand about Trigonometric ratios of complementary angles:
   a. $\sin (90^\circ - \theta) = \cos \theta$
   b. $\cos (90^\circ - \theta) = \sin \theta$
   c. $\tan (90^\circ - \theta) = \cot \theta$
   d. $\cosec (90^\circ - \theta) = \sec \theta$
   e. $\sec (90^\circ - \theta) = \cosec \theta$
   f. $\cot(90^\circ - \theta) = \tan \theta$. 

...
5. To know about Trigonometric Ratios of some specific angles and to use for solving sums.

<table>
<thead>
<tr>
<th>degrees</th>
<th>radians</th>
<th>sin θ</th>
<th>cos θ</th>
<th>tan θ</th>
<th>csc θ</th>
<th>sec θ</th>
<th>cot θ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>∞</td>
</tr>
<tr>
<td>30°</td>
<td>π/6</td>
<td>1/2</td>
<td>√3/2</td>
<td>√3</td>
<td>√3</td>
<td>2</td>
<td>√3</td>
</tr>
<tr>
<td>45°</td>
<td>π/4</td>
<td>√2/2</td>
<td>√2/2</td>
<td>1</td>
<td>√2</td>
<td>√2</td>
<td>1</td>
</tr>
<tr>
<td>60°</td>
<td>π/3</td>
<td>√3/2</td>
<td>1/2</td>
<td>√3</td>
<td>2√3</td>
<td>2</td>
<td>√3</td>
</tr>
<tr>
<td>90°</td>
<td>π/2</td>
<td>1</td>
<td>0</td>
<td>—</td>
<td>1</td>
<td>—</td>
<td>0</td>
</tr>
</tbody>
</table>

6. To prove and verify trigonometric identities

**CONCEPT MAP**
GRADED EXERCISES

LEVEL 1

1. If sec θ – tan θ = 1/3, then find the value of (sec θ + tan θ). (Ans. 3) (1 mark)
2. Find the value of tan 1°.tan 2°…tan 89°. (Ans. 1) (1 mark)
3. If Sin A = $\frac{1}{2}$, then find the value of Cos A (Ans. $\frac{\sqrt{3}}{2}$) (1 mark)
4. If Cos 9A = Sin A and 9A < 90°, then find the value of tan 5 A. (Ans 1) (1 mark)
5. If Sin A + Sin²A = 1, then find the value of Cos²A + Cos⁴A. (Ans.1) (1 mark)
6. If sec θ = $\frac{5}{4}$, then evaluate $\frac{\tan \theta}{1 + \tan^2 \theta}$. (Ans. $\frac{12}{25}$) (2 mark)
7. Prove that (tan A – tan B)^2 + (1 + tan A tan B)^2 = Sec²A Sec²B (2 mark)
8. Prove that $\frac{\cos \theta}{\cos \theta - 1} + \frac{\csc \theta}{\csc \theta + 1} = 2\sec^2 \theta$. (2 mark)
9. If 4 tan θ = 3, then evaluate $\tan^2 \theta - \csc^2 \theta$. (2 mark)
10. Prove that $\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = 2 \csc \theta$. (2 mark)

LEVEL 2

11. Prove that: $\cos^2 90° + \frac{\cos 80°}{\sin 10°} + \cos 59°. \csc 31° = 2$. (3 mark)
12. Prove that $\frac{\tan \theta - \cot \theta}{\sin \theta. \cos \theta} = \tan^2 \theta - \cot^2 \theta$ (3 mark)
13. If x = b Cos A – a Sin A and y = a Cos A + b Sin A, then prove that $x^2 + y^2 = a^2 + b^2$. (3 mark)
14. Prove that: $(\sin A + \csc A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$ (3 mark)
15. If Cot A = $\frac{3}{4}$, prove that $\frac{\sin A + \cos A}{\sin A - \cos A} = 7$ (3 mark)
16. Prove that $(\sin A + \cos A) (\tan A + \cot A) = \sec A + \cosec A$ (3 mark)
17. If 2 Sin²A – Cos²A = 2, then find the value of A (Ans. 90°) (3 mark)
18. Show that $\tan^4 \theta + \tan^2 \theta = \sec^4 \theta - \sec^2 \theta$. (3 mark)
19. A ladder 15 metres long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, find the height of the wall. (3 mark)
20. Simplify: $(1 + \tan^2 \theta) (1 - \sin \theta) (1 + \sin \theta)$ (3 mark)

LEVEL 3

21. $(\frac{3 \cos 45°}{\sin 47°})^2 - \frac{\cos 37° \cos 53°}{\tan 5°. \tan 25°. \tan 45°. \tan 65°. \tan 85°} = \cos 0°$ (Ans. 8) (4 mark)
22. If tan(A+B) =$\sqrt{3}$ and tan(A-B) =$\frac{1}{\sqrt{3}}$ and 0°< A+B <90°, A>B, Find A and B. (4 mark)
23. If \( \tan A = n \tan B \) and \( \sin A = m \sin B \) show that \( \cos^2 A = \frac{m^2 - 1}{n^2 - 1} \) (4 mark)

24. If \( \cos \theta + \sin \theta = \sqrt{2} \cos \theta \), then show that \( \cos \theta - \sin \theta = \sqrt{2} \sin \theta \). (4 mark)

25. If \( a \cos \theta + b \sin \theta = c \), then prove that \( a \cos \theta - b \sin \theta = \pm \sqrt{a^2 + b^2 - c^2} \) (4 mark)

26. If \( \sin \theta + \cos \theta = \sqrt{2} \), then prove that \( \tan \theta + \cot \theta = 1 \) (4 mark)

27. Prove that \( \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} = \tan \theta \). (4 mark)

28. Prove that \( \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \csc A + \cot A \) (4 mark)

29. If \( \csc \theta - \sin \theta = 1 \) and \( \sec \theta - \cos \theta = m \), prove that: \( 1^2 + m^2 + 3 = 1 \) (4 mark)

30. If \( \tan A + \sin A = m \) and \( \tan A - \sin A = n \), then show that \( m^2 - n^2 = 4 \sqrt{m \cdot n} \). (4 mark)

**VALUE BASED QUESTIONS**

1. If \( x = a \cos \theta \), \( y = b \sin \theta \), then \( b^2 x^2 + a^2 y^2 - a^2 b^2 \) is equal to zero; Is the statement correct? Give reasons for your answer. Which values are depicted here?

2. A Mathematics Teacher asked 'what is the value of \( x \) if \( \tan x = \sin 45^\circ \cos 45^\circ + \sin 30^\circ ? \) Sushmita answered 1, Sita says, ‘She is not correct.’ Give the correct answer to Sushmita. Which values of Sita area depicted here?

   Ans. \( x = 45^\circ \), Knowledge, curiosity.

3. What are the basic trigonometric identities. Also explain the values of identities?

4. Prove that \( \sin 45^\circ = 1 \). What value will you conclude?

5. Prove that \( \sin^2 47 + \cos^2 43 = 1 \). Find the value which can be extracted.

**ERROR ANALYSIS**

<table>
<thead>
<tr>
<th>Sl no</th>
<th>POSSIBLE ERRORS</th>
<th>REMEDIATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Writing ( \sin 90^\circ - \theta ) instead of ( \sin(90^\circ - \theta) )</td>
<td>It has to be emphasized that for all complementary angles it has to be written ( \sin (90^\circ - \theta) ) with in brackets</td>
</tr>
<tr>
<td>2</td>
<td>Some students draw other types of triangles instead of right angled triangles for calculating trigonometric ratios</td>
<td>Instruct the students that only right angled triangles are to be used and hypotenuse is related to right triangles only</td>
</tr>
<tr>
<td>3</td>
<td>Some students do not mention the angle along with the trigonometric ratio ( \tan = \frac{\sin}{\cos} )</td>
<td>Emphasize on writing ratio with correct angles and give sufficient practice.</td>
</tr>
<tr>
<td>4</td>
<td>Students consider ( \sin^2 \theta + \cos^2 \theta = 1 ) and take its square root as ( \sin \theta + \cos \theta = \pm 1 )</td>
<td>Instruct the students that square root is taken only for whole term i.e. not with + and - sign</td>
</tr>
<tr>
<td>5</td>
<td>Use of trigonometric ratios to prove geometrical results, is not very common with students while this method becomes very useful in some of the questions</td>
<td>Students should be encouraged to use trigonometric results in geometry, especially where the ratio of sides is given or the angles are ( 30^\circ, 45^\circ, 60^\circ )</td>
</tr>
</tbody>
</table>
QUESTION BANK

1. If $\cos^3 A + 3\cos A \sin^2 A = m$ and $\sin^3 A + 3\cos A \sin^2 A = n$, then prove that $(m+n)^{2/3} + (m - n)^{2/3} = 2a^{2/3}$.

2. Solve for $\phi$ if \[
\frac{\sin \phi}{1 + \cos \phi} + \frac{1 + \cos \phi}{\sin \phi} = 4.
\] [Ans: $\phi = 30^\circ$]

3. If \[
\frac{1 - \tan \theta}{1 + \tan \theta} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1},
\] show that \[
\frac{\sin \theta}{\cos 2\theta} = 1.
\]

4. If $a\cos \Theta - b\sin \Theta = c$, prove that $a\sin \Theta + b\cos \Theta = \sqrt{(a^2 + b^2 - c^2)}$.

5. If $\cos \phi + \sin \phi = \sqrt{2} \cos \phi$, prove that $\cos \phi - \sin \phi = \sqrt{2} \sin \phi$.

6. Prove that \[
2(\cos^6 \phi + \sin^6 \phi) - 3(\cos^4 \phi + \sin^4 \phi) + 1 = 0.
\]

7. If $\sec \theta - \tan \theta = 4$, then prove that $\cos \theta = \frac{9}{17}$.

8. Prove that \[
\frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} = \frac{1 + \sin \theta}{\cos \theta}.
\]

9. Prove that \[
\sqrt{\sec \theta - 1} + \sqrt{\sec \theta + 1} = 2 \csc \theta.
\]

10. If $\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$ and $\frac{x}{a} \sin \theta - \frac{y}{b} \cos \theta = 1$, prove that $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$. 

ACTIVITIES/PROJECTS

1. To make a mathematical instrument ‘clinometer’ to measure the height of a distant object.
2. To make a model to show angle of elevation of any given object.
3. To make a model to show angle of depression of any given object.
5. To find the height of a light post sby using Basic Proportionality Theorem.

POWER POINT PRESENTATIONS

WEB LINKS

- Trigonometry! Simple Hand Trick for Memorizing Values
  - https://www.youtube.com/watch?v=xXGfp9PKdXM
- Videos on Trigonometry
  - https://www.youtube.com/watch?v=nIblk99ji0c
- Videos on Trigonometry
  - https://www.youtube.com/watch?v=Jsity4TxlIME
- Hand trick to make formula easy web site
  - http://www.meritnation.com/maths.htm?mncid=Adwords_Restr._Math&gclid=CMTvpqrquisCFT0AmA
- How to remember trigonometric rasly
  - http://www.topperlearning.com/study/cbse/class-10/mathematics/chapter/introduction-to-trigonometry/b101c2s3ch89?gclid=CLX3-Kavis
- Basic trig function
- Introduction to trigonometry Videos
  - https://www.google.co.in/search?q=trigonometric+ratios+of+complementary+angles&tbm=isch&imgil=f_0KANgWDyajzM%253A%253A
CHAPTER-14
STATISTICS
INTRODUCTION

Measures of central tendency refer to all those methods of statistical analysis by which averages of the statistical series are worked out.

The word average is very commonly used in day-to-day conversation. However, in statistics the term average has a different meaning. It is defined as that value of a distribution, which is considered as the most representative value of the series. Since an average represents the entire data, its value lies between the largest and smallest item. For this reason, an average is frequently referred to as a measure of central tendency.

Different Measures Of Central Tendency

Arithmetic mean (simple and weighted)
Median
Mode

EXPECTED LEARNING OUTCOMES

1. To understand about the raw data, grouped data and ungrouped data.
2. To know about measures of central tendency.
3. To know how to calculate mean by
   (a) Direct Method
   (b) Assumed Mean Method
   (c) Step Deviation Method.
4. To compute the median of a given data and to plot the graphs of less than Ogive and More than Ogive.
5. To compute mode of the given data
**GRADED EXERCISES**

**LEVEL 1**

1. The median and mean of a frequency distribution are 24 and 28 respectively. Find the mode. (Ans. 16) (1 MARK)

2. What measure of central tendency is represented by the abscissa of the point where less than Ogive and more than Ogive intersect? (Ans. Median) (1 mark)

3. Find the mode class and the median class for the following distribution:
4. Calculate the mode of the following data

<table>
<thead>
<tr>
<th>Class Interval</th>
<th>0 - 10</th>
<th>10 – 20</th>
<th>20 – 30</th>
<th>30 – 40</th>
<th>40 – 50</th>
</tr>
</thead>
<tbody>
<tr>
<td>frequency</td>
<td>6</td>
<td>10</td>
<td>12</td>
<td>8</td>
<td>7</td>
</tr>
</tbody>
</table>

(Ans. Modal class and median class is 20 – 30) (2 mark)

5. If the mean of the following data is 18.75, find the value of p

<table>
<thead>
<tr>
<th>x</th>
<th>10</th>
<th>15</th>
<th>P</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>f</td>
<td>5</td>
<td>10</td>
<td>7</td>
<td>8</td>
<td>2</td>
</tr>
</tbody>
</table>

(Ans. P = 20) (2 mark)

LEVEL2

6. The mean of the following distribution is 62.8 find the missing frequency $F_1$ and $F_2$

<table>
<thead>
<tr>
<th>Class Interval</th>
<th>0 - 20</th>
<th>20 - 40</th>
<th>40 - 60</th>
<th>60 - 80</th>
<th>80 - 100</th>
<th>100 - 120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>5</td>
<td>$F_1$</td>
<td>10</td>
<td>$F_2$</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

(Ans. $F_1 = 8$, $F_2 = 12$) (3 mark)

7. Find the median of the following table:

<table>
<thead>
<tr>
<th>Class</th>
<th>100 - 120</th>
<th>120 – 140</th>
<th>140 - 160</th>
<th>160 – 180</th>
<th>180 – 200</th>
</tr>
</thead>
<tbody>
<tr>
<td>frequency</td>
<td>12</td>
<td>14</td>
<td>8</td>
<td>6</td>
<td>10</td>
</tr>
</tbody>
</table>

(Ans. 138.57) (3 mark)

8. Draw ‘less than’ ogive curve for the following distribution:

<table>
<thead>
<tr>
<th>Class interval</th>
<th>20 – 30</th>
<th>30 - 40</th>
<th>40 – 50</th>
<th>50 - 60</th>
<th>60 70</th>
<th>70 - 80</th>
<th>80 - 90</th>
</tr>
</thead>
<tbody>
<tr>
<td>frequency</td>
<td>10</td>
<td>8</td>
<td>12</td>
<td>24</td>
<td>6</td>
<td>25</td>
<td>15</td>
</tr>
</tbody>
</table>

(3 mark)

9. Find the mode of the following table shows the ages of the patients admitted in a hospital during a year:

<table>
<thead>
<tr>
<th>Age (in years)</th>
<th>5 - 15</th>
<th>15 - 25</th>
<th>25 - 35</th>
<th>35 - 45</th>
<th>45 - 55</th>
<th>55 - 65</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Patients</td>
<td>6</td>
<td>11</td>
<td>21</td>
<td>23</td>
<td>14</td>
<td>5</td>
</tr>
</tbody>
</table>

(3 mark)

10. Draw ‘more than’ ogive curve for the following distribution:

<table>
<thead>
<tr>
<th>Class interval</th>
<th>20 – 30</th>
<th>30 - 40</th>
<th>40 – 50</th>
<th>50 - 60</th>
<th>60 70</th>
<th>70 - 80</th>
<th>80 - 90</th>
</tr>
</thead>
<tbody>
<tr>
<td>frequency</td>
<td>10</td>
<td>8</td>
<td>12</td>
<td>24</td>
<td>6</td>
<td>25</td>
<td>15</td>
</tr>
</tbody>
</table>

(3 mark)
11. The following table shows the data of the amount donated by 100 people in a blind school.

<table>
<thead>
<tr>
<th>Amount Donated (in Rs.)</th>
<th>Number of persons</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 100</td>
<td>2</td>
</tr>
<tr>
<td>100 - 200</td>
<td>5</td>
</tr>
<tr>
<td>200 - 300</td>
<td>X</td>
</tr>
<tr>
<td>300 - 400</td>
<td>12</td>
</tr>
<tr>
<td>400 - 500</td>
<td>17</td>
</tr>
<tr>
<td>500 - 600</td>
<td>20</td>
</tr>
<tr>
<td>600 - 700</td>
<td>Y</td>
</tr>
<tr>
<td>700 - 800</td>
<td>9</td>
</tr>
<tr>
<td>800 - 900</td>
<td>7</td>
</tr>
<tr>
<td>900 - 1000</td>
<td>4</td>
</tr>
</tbody>
</table>

If the median of the above data is 525, find the value of X and Y. What values are depicted here? (Ans. \( x = 9, y = 15 \)) (4 mark)

12. The following table gives production yield per hectare of wheat of 100 farms of a village

<table>
<thead>
<tr>
<th>Production yield (in Kg/ha)</th>
<th>50 - 55</th>
<th>55 - 60</th>
<th>60 - 65</th>
<th>65 - 70</th>
<th>70 - 75</th>
<th>75 - 80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of farms</td>
<td>2</td>
<td>8</td>
<td>12</td>
<td>24</td>
<td>38</td>
<td>16</td>
</tr>
</tbody>
</table>

Draw ‘more than ogive’ and less than ogive. What is the median obtained. (4 mark)

13. The mean of the following frequency distribution is 57.6 and the sum of the observation is 50 Find the missing frequencies: \( f_1 \) and \( f_2 \).

<table>
<thead>
<tr>
<th>Class</th>
<th>0 - 20</th>
<th>20 - 40</th>
<th>40 - 60</th>
<th>60 - 80</th>
<th>80 - 100</th>
<th>100 - 120</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>7</td>
<td>( f_1 )</td>
<td>12</td>
<td>( f_2 )</td>
<td>8</td>
<td>5</td>
<td>50</td>
</tr>
</tbody>
</table>

(Ans. \( f_1 = 8 \), \( f_2 = 10 \)) (4 mark)

14. Draw less than ogive curve and find the median of the following frequency distribution table:
15. If the median of the following data is 32.5. Find the value of x and y:

<table>
<thead>
<tr>
<th>Class Interval</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 10</td>
<td>x</td>
</tr>
<tr>
<td>10 - 20</td>
<td>5</td>
</tr>
<tr>
<td>20 - 30</td>
<td>9</td>
</tr>
<tr>
<td>30 - 40</td>
<td>12</td>
</tr>
<tr>
<td>40 - 50</td>
<td>y</td>
</tr>
<tr>
<td>50 - 60</td>
<td>3</td>
</tr>
<tr>
<td>60 - 70</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td>40</td>
</tr>
</tbody>
</table>

**VALUE BASED QUESTIONS**

1. Every year Vivekananda public school in Hyderabad collecting some possible contributions voluntarily with school children

<table>
<thead>
<tr>
<th>Age group of people</th>
<th>Contributions (in Rs ten thousands)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 - 30</td>
<td>50</td>
</tr>
<tr>
<td>30 - 35</td>
<td>100</td>
</tr>
<tr>
<td>35 - 40</td>
<td>150</td>
</tr>
<tr>
<td>40 - 45</td>
<td>200</td>
</tr>
</tbody>
</table>

(a) In this contribution 50% money is contributed to handicapped

(b) 25% old age people

(c) 25% cyclone relief

Draw a less than ogive. (This reflects courtesy towards nation)

2. The monthly Saving of a certain households of a locality is given in the following table:

<table>
<thead>
<tr>
<th>Monthly Savings in Rs.</th>
<th>1000 - 2000</th>
<th>2000 – 3000</th>
<th>3000 – 4000</th>
<th>4000 - 5000</th>
<th>5000 - 6000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of House holds</td>
<td>8</td>
<td>15</td>
<td>10</td>
<td>12</td>
<td>5</td>
</tr>
</tbody>
</table>

Find the average monthly saving.

What values do these households possess?

3. Some people of a society decorated their area with flags and tricolor ribbons on Republic Day. The following data shows the number of persons in different age group who participated in the decoration.
<table>
<thead>
<tr>
<th>Age in years</th>
<th>5 - 15</th>
<th>15 - 25</th>
<th>25 - 35</th>
<th>35 - 45</th>
<th>45 - 55</th>
<th>55 - 65</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of patients</td>
<td>6</td>
<td>11</td>
<td>21</td>
<td>23</td>
<td>14</td>
<td>5</td>
</tr>
</tbody>
</table>

Find the mode of the above data.
What values do these persons possess?

4. A group of people planned for community service for which they selected a cancer institute and organized some activities for their entertainment. They collected the following data from the hospital:

<table>
<thead>
<tr>
<th>Age in years</th>
<th>5 - 15</th>
<th>15 - 25</th>
<th>25 - 35</th>
<th>35 - 45</th>
<th>45 - 55</th>
<th>55 - 65</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of patients</td>
<td>6</td>
<td>11</td>
<td>21</td>
<td>23</td>
<td>14</td>
<td>5</td>
</tr>
</tbody>
</table>

Convert the above distribution into a ‘less than type’ cumulative frequency distribution and draw its ogive. Also find the median from this ogive.

What values do these persons possess?

5. The amounts donated by some households in their religious organisations are as follows:

<table>
<thead>
<tr>
<th>Amount in Rs.</th>
<th>Number of households</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than 100</td>
<td>14</td>
</tr>
<tr>
<td>Less than 200</td>
<td>22</td>
</tr>
<tr>
<td>Less than 300</td>
<td>37</td>
</tr>
<tr>
<td>Less than 400</td>
<td>58</td>
</tr>
<tr>
<td>Less than 500</td>
<td>67</td>
</tr>
<tr>
<td>Less than 600</td>
<td>75</td>
</tr>
</tbody>
</table>

Calculate the arithmetic mean for the above data.
What values do these households possess?
ERROR ANALYSIS

<table>
<thead>
<tr>
<th>Sno</th>
<th>POSSIBLE ERRORS</th>
<th>REMEDIATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mean of the data is calculated incorrectly</td>
<td>Emphasize on the concept that mean = \frac{\text{sum of observations}}{\text{Number of Observations}} and give enough practice.</td>
</tr>
<tr>
<td>2</td>
<td>Calculate mode and median for grouped data</td>
<td>Students should ensure class intervals are continuous.</td>
</tr>
<tr>
<td>3</td>
<td>Construction of ogive.</td>
<td>Students should ensure class intervals are continuous.</td>
</tr>
</tbody>
</table>

QUESTION BANK

1. The mean of 30 numbers is 18, what will be the new mean, if each observation is increased by 2? (Ans: 20)

2. Find the mean of 30 numbers given mean of ten of them is 12 and the mean of remaining 20 is 9. (Ans: 10)

3. The average weight of students in 4 sections A, B, C and D is 60 kg. The average weights of the students of A, B, C and D individually are 45kg, 50kg, 72kg and 80kg respectively. If the average weight of the students of section A and B together is 48 kg and that of the students of B and C together is 60 kg, what is the ratio of the number of students in section A and D? (Ans: 4:3)

4. The mode of a distribution is 55 & the modal class is 45-60 and the frequency preceding the modal class is 5 and the frequency after the modal class is 10. Find the frequency of the modal class. (Ans: 15)

5. The Median and Mode of the following wage distribution are known to be Rs.33.5 and Rs.34 respectively. Three frequency values from the table, however are missing. Find the missing frequencies. [Ans: x = 950, y = 100, z = 40]

6. Find the median of the following data:

<table>
<thead>
<tr>
<th>Marks obtained</th>
<th>0 - 10</th>
<th>10 – 20</th>
<th>20 – 30</th>
<th>30 – 40</th>
<th>40 – 50</th>
<th>50 – 60</th>
<th>60 – 70</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.of Students</td>
<td>6</td>
<td>5</td>
<td>12</td>
<td>22</td>
<td>15</td>
<td></td>
<td></td>
<td>60</td>
</tr>
</tbody>
</table>

Ans: Median = 46.67

7. Find the mode of the following distribution of marks obtained by 60 students

<table>
<thead>
<tr>
<th>Marks obtained</th>
<th>0 - 10</th>
<th>10 – 20</th>
<th>20 - 30</th>
<th>30 - 40</th>
<th>40 - 50</th>
<th>40 - 50</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.of Students</td>
<td>6</td>
<td>5</td>
<td>12</td>
<td>22</td>
<td>15</td>
<td></td>
</tr>
</tbody>
</table>

8. Following distribution shows the marks obtained by the class of 100 students,

<table>
<thead>
<tr>
<th>Marks</th>
<th>10 - 20</th>
<th>20 - 30</th>
<th>30 - 40</th>
<th>40 - 50</th>
<th>50 - 60</th>
<th>60 - 70</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.of Students</td>
<td>10</td>
<td>15</td>
<td>30</td>
<td>32</td>
<td>8</td>
<td>5</td>
</tr>
</tbody>
</table>

Draw less than ogive for the above data. Find median graphically and verify the result by actual method (Ans: 38.33)
ACTIVITIES/PROJECTS

1. Analysis of language text using graphs.
2. To find average height of students studying in class X
3. Find the median height of your class students height by drawing less than ogive and greater than ogive. Also find the median height & verify the median height of the student by the formula
4. Find the median weight of the student in your class.

POWER POINT PRESENTATIONS

WEB LINKS

- CBSE Class X Statistics information
- Statistics basic definition
- https://www.youtube.com/watch?v=-ukY6HCWcT8
- mathsjournels
- Mean of grouped Data Web Site
- Median of grouped data
- Mean, Median & mode watch videos.
- https://www.youtube.com/watch?v=lxkpP4VYIDc
Resources – Chapter wise

TERM -2
CHAPTER-4

QUADRATIC EQUATIONS

INTRODUCTION

Children are familiar with linear equations in one variable and their solutions. Also, they studied what are linear, quadratic and cubic polynomials. Recall a quadratic polynomial in a single variable ‘x’, $ax^2 + bx + c$, and their solution.

When a polynomial is equated to zero, we get an equation. Thus, $ax^2 + bx + c = 0$ where $a, b, c$ are real numbers and $a \neq 0$ is a quadratic equation in one variable $x$. If $ax^2 + bx + c = 0$, for $x = a$, then $a$ is called the root of the equation.

Such equations arise in many real life situations. In this chapter, we learn about quadratic equations and various ways of finding their roots. At the end we will discuss to represent the given situation in the form of quadratic equations and solve.

Solving of quadratic equations is often credited to ancient mathematicians. Brahmagupta (A.D.598-665) gave an explicit formula to solve a quadratic equation of the form $ax^2 + bx = c$. Later Sridharacharya (A.D.1025) derived a formula, now known as quadratic formula, for solving a quadratic equation by the method of completing the square.

Expected Learning Outcomes

1. To identify the general form of a Quadratic Equation.
2. To understand the meaning of roots of a quadratic equation.
3. To understand the methods of solving quadratic equation.
   a) Factorisation method
   b) Completing the square method
   c) Using quadratic formula.
4. To understand the nature of roots of a quadratic equation.
5. Framing quadratic equation from a given word problem and solving it.
CONCEPT MAP

QUADRATIC EQUATIONS

General form

Solution

Factorisation

Completing square

Quadratic formula

Nature of roots depends on Discriminant

D= 0, Equal roots

D > 0, Real distinct roots

D < 0, No real roots

Quad.Eqns.vue
GRADED EXERCISES

**LEVEL-1**

1. Check whether \( x = (-1) \) is a solution of equation \( 4x^2 - 3x - 1 = 0 \) (1mark)
2. Find \( k \), if one root of equation \( x^2 + kx - 4 = 0 \) (1mark)
3. Solve by factorization: \( 9x^2 - 3x - 20 = 0 \) (2marks)
4. Solve by completing square method: \( 6x^2 - 13x - 5 = 0 \) (2marks)
5. Find the discriminant of the equation: \( 2x^2 - 7x + 3 = 0 \) (1mark)
6. Find the nature of roots of equation \( 9x^2 + 12x + 4 = 0 \) (1mark)
7. Find \( k \), if \( 2kx^2 + 6x + 5 = 0 \) has equal roots. (2marks)
8. \( \frac{x - 1}{x + 1} = 3 \) (x ≠ 0) (2marks)
9. The sum of roots of the equation \( 2x^2 + 7x - 4 \) is ………………. (1mark)
10. The product of roots of the equation \( 2x^2 + 7x - 4 \) is ………………. (1mark)
11. If 2 is a root of the equation \( x^2 + bx + 12 = 0 \), find the value of ‘b’ and find the other root. (2marks)

**LEVEL-2**

1. Find the value of \( k \), for which the quadratic equation \( (k-12) x^2 + 2(k-12)x + 2 = 0 \) has equal roots. (2marks)
2. Find the of roots of the equation: (i) \( 4x^2 + 4\sqrt{3}x + 3 = 0 \) (2marks)
   (ii) \( 4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0 \) (2marks)
3. The product of 2 consecutive integers is 182. Find the integers. (3marks)
4. Solve by factorisation: \( 3x^2 - 2\sqrt{6}x + 2 = 0 \). (2marks)
5. Find the roots of the equation \( 3x^2 + 6x + 1 = 0 \) by method of completing perfect square. (3marks)
6. If \((-5)\) is a root of the quadratic equation \( 2x^2 + px - 15 = 0 \) and the quadratic equation \( p(x^2 + x) + k = 0 \) has equal roots, find the value of \( p \) and \( k \). (3marks)
7. Find the value of \( p \) so that the equation \( px(x-3) + 9 = 0 \) has equal roots. (2marks)
8. Find the roots of the following equation:
   \[ \frac{1}{x + 4} - \frac{1}{x - 7} = \frac{11}{30} \] (x ≠ -4,7) (3marks)

**LEVEL 3**

1. Solve: \( \frac{1}{a + b + x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \) \( (a ≠ 0, b≠ 0, x≠ 0) \) (3marks)
2. Using quadratic formula, solve: (i) \( 3a^2x^2 + 8abx + 4b^2 = 0 \) \( (a ≠ 0) \) (3marks)
   (ii) \( 9x^2 - 3(a + b)x + ab = 0 \) (3marks)
   (iii) \( p^2x^2 + (p^2 - q^2)x - q^2 = 0 \) (3marks)
3. Solve: \( \frac{x + 1}{x - 1} + \frac{x - 2}{x + 2} = 3 \) (\( x \neq 1, -2 \)) (3 marks)

4. Solve by factorisation: \( a^2x^2 - (a^2b^2 + 1)x + b^2 = 0 \) (3 marks)

5. Solve: \( \frac{x - a}{x - b} + \frac{x - b}{x - a} = \frac{a}{b} + \frac{b}{a} \) (3 marks)

6. The speed of a boat in still water is 11 km/hr. It can go 12 km upstream and return downstream to the original point in 2 hours 45 minutes. Find the speed of stream. (4 marks)

7. A train travels at a uniform speed for a distance of 63 km and then travels a distance of 72 km at an average speed of 6 km/hr more than its original speed. If it takes 3 hours to complete the total journey, what is the original speed of the train? (4 marks)

8. The sum of the areas of two squares is 640 m\(^2\). If the difference of their perimeters is 64 m, find the sides of the two squares. (24 m, 8 m) (4 marks)

9. The difference of two numbers is 5 and the difference of their reciprocal is \( \frac{1}{10} \). Find the numbers. (4 marks)

10. In a flight of 2800 km, an aircraft was slowed down due to bad weather. Its average speed is reduced by 100 km/hr and time increased by 30 minutes. Find the original duration of flight. (4 marks)

**VALUE BASED QUESTIONS**

1. Rs. 9000 were divided equally among a certain number of students. Amit was given the responsibility of dividing this amount among the students but 20 more students admitted to the school. Now each student got Rs. 160 less. Find the original number of students. What value of Amit is depicted in the question?

2. If the price of petrol is increased by Rs. 2 per litre, a person will have to buy 1 litre less petrol for Rs. 1740. Find the original price of petrol at that time.
   (i) Why do you think the price of petrol is increasing day by day?
   (ii) What should we do to save petrol?

3. Due to some technical problems, an aeroplane started late by one hour from its starting point. The pilot decided to increase the speed of the aeroplane by 100 km/hr from its usual speed, to cover a journey of 1200 km in time. Find the usual speed of the aeroplane? What value (Quality) of the pilot is represented in the question?

4. In the centre of a rectangular plot of land of dimensions 120 m × 100 m, a rectangular portion is to be covered with trees so that the area of remaining part of the plot is 10500 m\(^2\).
   (i) Find the dimensions of the area to be planted.
   (ii) Which social act is being discussed here? Give its advantages.

5. Mr. Arun has two square plots of land which he utilizes for two different purposes - one for providing free education to the children below the age of 14 years and the other to provide free medical services to the needy villagers. The sum of the areas of two square plots is 15425 m\(^2\). If the difference of their perimeters is 60 m, find the sides of the two squares. Which qualities of Mr. Arun are being depicted here?
## ERROR ANALYSIS

<table>
<thead>
<tr>
<th>Sl no</th>
<th>POSSIBLE ERRORS</th>
<th>REMEDIATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>In problems on Speed, distance and time the students do not know to frame the equation - when speed is increased, time taken is less and vice versa</td>
<td>More practice is to be given for such type of sums</td>
</tr>
<tr>
<td>2</td>
<td>While finding the discriminant students take the value of a and b with variables</td>
<td>Practice to be given with the knowledge of coefficient of x^2 is a and coefficient of x is b</td>
</tr>
<tr>
<td>3</td>
<td>Questions where two consecutive even integers given- students write as x and x+1</td>
<td>Emphasize that difference between two consecutive integers is 2. So we should take x and x+2</td>
</tr>
<tr>
<td>4</td>
<td>Questions on difference between 2 positive integers and difference between their reciprocals. Students write as x-y =….. and ( \frac{1}{x} - \frac{1}{y} = \ldots )</td>
<td>Emphasis should be given when x&gt;y, ( \frac{1}{x} &lt; \frac{1}{y} ) and will be negative.</td>
</tr>
</tbody>
</table>

## QUESTION BANK

1. A natural number when increased by 12 equals 160 times of its reciprocal. Find the number.
2. Find a natural number whose square diminished by 84 is equal to thrice of 8 more than the given number.
3. If the equation \( (1+ m^2) x^2 + 2mcx + c^2 - a^2 = 0 \), has equal roots, show that At present \( c^2 = a^2(1+ m^2) \)
4. At present Asha’s age is 2 more than the square of her daughter Nisha’s age. When Nisha grows to her mother’s present age, Asha’s age would be one year less than 10 times the present age of Nisha. Find the present ages of Asha and Nisha.
5. The length of a rectangle is thrice as long as the side of a square. The side of the square is 4 cm more than the width of the rectangle. If their areas being equal, find their dimensions.
ACTIVITIES/PROJECTS

1. To solve a quadratic equation by the method of completion of squares.
   Project: Report on algebra in daily life

POWER POINT PRESENTATIONS

WEB LINKS

Factorization

https://www.youtube.com/watch?v=ZQ-NRsWhOG

Nature of roots

https://www.youtube.com/watch?v=Hh6B362fIbk

Completing square method

https://www.youtube.com/watch?v=xGOQYTo9AKY
CHAPTER-5

ARITHMETIC PROGRESSIONS

INTRODUCTION

In nature, many things follow a certain pattern, such as the petals of a sunflower, the holes of a honeycomb, the spirals on a pineapple etc. Articles of same kind we generally arrange according to their sizes either in ascending order or descending order. We come across some patterns which occur in day-to-day life such as

(i) a person’s salary with the annual increment

(ii) Money deposited in a bank with the simple interest

(iii) The lengths of the rungs of a ladder decrease uniformly from top to bottom.

In this chapter, we discuss one of the patterns in which the succeeding terms are obtained by adding a fixed number to the preceding terms – called Arithmetic progressions.

By the end of this chapter, children should be able to cite examples of A.P, appreciate the pattern in numbers. They should be able to form an A.P in a situational problem, find the n\text{th} term and sum of ‘n’ terms.

Expected Learning Outcomes

1. To understand the concept of an Arithmetic progression.
2. To identify terms and common difference in an A.P.
3. To find the n\text{th} term of an A.P- a_n
   a) To find number of terms ‘n’
   b) To find the common difference ‘d’
   c) To find the first term ‘a’
   d) To check whether a number is a term of given A.P.
4. To find the A.P. when two terms are given.
5. To find the sum of ‘n’ terms of an A.P.
6. To find a_n when the general term S_n is given.
7. To form an A.P. from a given situational problem and solve.
GRADED EXERCISES

LEVEL--1

1. Is the list of numbers 3, 9, 15, 21…………………..form an A.P? (1mark)
2. Find the first term and common difference of the A.P: 1, 5, 9, 13,17……………… (1mark)
3. In the given A.P. find the missing term: \( \sqrt{2}, \ldots, 5\sqrt{2} \) (1mark)
4. Find the 15\(^{th}\) term of an A.P: 24, 21, 18………………….. (1mark)
5. Find the value of x if 3x-4, 4x-7, 7x-3 are in A.P.(1mark)
6. The first term of an AP is 2 and common difference is 4, then find its 31\(^{st}\) term . (1mark)
7. The common difference of an A.P. is 5, then find the value of \(a_9 - a_5\) (1mark)
8. If \(a_n = 5 - 3n\) for an A.P, find the common difference. (1mark)
9. Is 68 a term of the AP: 7, 10, 13,…. ? (2marks)
10. Find the 5\(^{th}\) term from the end of an A.P… -11, -8, -5…………………..-49. (2marks)
11. Find the 31\(^{st}\) term of an A.P. whose 11\(^{th}\) term is 38 and 16\(^{th}\) term is 73. (2marks)
12. Find the sum of 24 terms of the A.P: 5, 8, 11, 14, ………………….  .  
13. Find the sum of first 15 multiples of 8.  

**LEVEL-II**

1. Which term of the A.P. 84, 80, 76 ………………. is zero?  
2. Find the sum of odd numbers between 0 and 50.  
3. Which term of the sequence 48, 43, 38, 33……………. is the first negative term?  
4. Which term of the A.P. 4, 12, 20, 28,… will be 120 more than its 21st term?  
5. Find the 20th term of an AP whose 7th term is 24 less than the 11th term, first term being 12.  
6. Determine the AP whose 5th term is 15 and sum of its 3rd and 8th term is 34.  
7. Find the A.P. whose 3rd term is16 and the difference of 5th term and 7th term is12.  
8. Find the sum of all 3- digit numbers less than 300 which are divisible by 7.  
9. The first term of an AP is 100 and last term is (-10). If the common difference is (-2), how many terms are there and what is their sum?  
10. The sum of 14 terms of an A.P is 1050 and its first term is 10. Find its 30th term.  
11. In an A.P. the first term is 2 and the last term is 29. The sum of the terms is 155. Find the common difference.  
13. Find the sum of first 51 terms of an AP whose 2nd and 3rd terms are 14 and 18 respectively.  
14. Find the sum of 15 terms of an A.P. whose nth term is given by 9 – 5n.  

**LEVEL-III**

1. The sum of the 5th and the 9th terms of the AP is 30. If its 25th term is three times its 8th term, find the AP.  
2. How many terms of an AP 17, 15, 13, 11…. must be added to get the sum 72? Explain the double answer.  
3. Find the sum of integers between 100 and 700 which on dividing by 11 leave a remainder 7.  
4. The sum of n terms of an AP is given by $5n^2 - 3n$. Find the AP and its 20th term.  
5. The sum of first 15 terms of an A. P. is 105 and the sum of next 15 terms is 780. Find the first 3 terms of the A. P.  
6. If 7 times the seventh term of an A.P. is equal to 11 times the 11th term, show that its 18th term is equal to zero.  
7. If the sum of first n terms of an A.P. is given by $S_n = 4n^2 - 3n$, find the nth term of the A.P.  
8. Find the value of middle term of the A.P: -6, -2, 2, 6…………………58.  
9. The 19th term of an A.P. is equal to 3 times its 6th term. If the 9th term of the A.P. is 19, find the A.P.  
10. The sum of 3 numbers of an A.P. is 3 and their product is (-35). Find the numbers.  
11. The sum of first 3 terms an A.P is 15. If the sum of their squares is 93, find the A.P.
12. The sum of first 6 terms of an A.P is 42. The ratio of its 10th term to its 30th term is 1:3.
   Calculate the first and 13th term of the A.P. (4marks)

**VALUE BASED QUESTIONS**

1. Ram asks the labour to dig a well up to a depth of 10 metre. Labour charges Rs.150 for first metre and Rs. 50 for each subsequent metres. As labour was uneducated, he claims Rs. 550 for the whole work. What should be the actual amount to be paid to the labour? What value of Ram is depicted in the question if he pays Rs. 600 to the labour?

2. Nidhi saves Rs. 2 on first day of the month, Rs. 4 on second day, Rs. 6 on third day and so on. What will be her saving in the month of Feb. 2012? What value is depicted by Nidhi?

3. How many two digit numbers are there in between 6 and 102 which are divisible by 6. Ram calculated it by using A.P. while Shyam calculated it directly. Which value is depicted by Ram?

4. In a school, students thought of planting trees in an around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying. e.g. a section of class-I will plant 1 tree, a section of class-II will plant 2 trees and so on till class XII. There are three sections of each class. How many trees will be planted by the students? What value can you infer from the planting the trees?

5. A person donates money to a trust working for education of children and women in some villages. If the person donates Rs. 5000 in the first year and his donation increases by Rs. 250 every year, find the amount donated by him in the eighth year and the total amount donated in 8 years.
   Write any two values the person possesses.
   Why do you think education of women is necessary for the development of society?

**ERROR ANALYSIS**

<table>
<thead>
<tr>
<th>Sl no</th>
<th>POSSIBLE ERRORS</th>
<th>REMEDIATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>When the numbers in A.P is in decreasing order, 7,4,1….. students write d = 3</td>
<td>Emphasis should be given on d = a₂ – a₁ = 4 - 7 = (-3)</td>
</tr>
<tr>
<td>2</td>
<td>When ‘d’ is negative, in questions to find ‘n’ Students do not multiply (n-1) with negative number and error occurs.</td>
<td>Put d in bracket. Ex: 63 = 3 + (n-1) (-3) so that multiplication will be taken up in the next step.</td>
</tr>
<tr>
<td>3</td>
<td>Confusion between aₙ and Sₙ</td>
<td>Sufficient practice must be given in solving problems so that the students are able to differentiate / recognize the type of problems</td>
</tr>
<tr>
<td>4</td>
<td>To find the nth term from the last of a given AP, students take the common difference of the given AP</td>
<td>Similar type of sums should be given practice</td>
</tr>
</tbody>
</table>

**QUESTION BANK**

1. The sum of first 5 terms of an A.P and the sum of first 7 terms of an A.P is 167. If sum of first 10 terms of the A.P is 235, find the sum of its first 20 terms.
2. Find the sum of first 7 numbers which are multiples of 2 as well as multiples of 9.
(Hint: Take the LCM of 2 and 9)

3. Solve: \((-4) + (-1) + 2+ \ldots \ldots + x = 437\)

4. The eighth term of an A.P. is half its second term and the eleventh term exceeds one third of its fourth term by 1. Find the fifteenth term.

5. The sum of 4 consecutive terms of an A.P. is 32 and the ratio of product of the first and last terms to the product of the two middle terms is 7:15. Find the numbers.
   (Hint: Take the 4 consecutive terms as a-3d, a-d, a+d, a+3d)

**ACTIVITIES**

1. To check whether a given sequence is an AP or not.
2. To find the formula for \(n\)th term of an arithmetic progression.
3. To find the formula for the sum of first “\(n\)” natural numbers.
4. To find the formula for the sum of first “\(n\)” odd natural numbers.

**PROJECTS**

(i) Find out from your surroundings, different illustrative situations where quantities progress arithmetically.

(i) Mathematical designs and patterns using AP

**POWER POINT PRESENTATIONS**

**WEB LINKS**

- Introduction
  [https://www.youtube.com/watch?v=1SDLmYdZkho](https://www.youtube.com/watch?v=1SDLmYdZkho)
- AP
  [https://www.youtube.com/playlist?list=PL5239D78BDA39CF2E](https://www.youtube.com/playlist?list=PL5239D78BDA39CF2E)
- sum of \(n\) \('terms"
  [https://www.youtube.com/watch?v=xhah3xwVtSU](https://www.youtube.com/watch?v=xhah3xwVtSU)
CHAPTER-7

COORDINATE GEOMETRY

INTRODUCTION

It was the work of French mathematician Rene Descartes (1596-1650) that merged algebra with geometry.

In coordinate system, every point (a geometric concept) is assigned a pair of numbers (an arithmetic concept) as its unique address, based on two real number lines intersecting at right angles.

In class IX, children have studied to locate the position of a point in a coordinate plane.

In this chapter, reinforce the plotting of points in a coordinate plane and discuss about

- how to find the distance between two points in the coordinate plane.
- how to find the coordinates of the point which divides a line segment joining two given points in a given ratio.
- calculate the ratio in which a point divides a line segment.
- how to find the area of the triangle formed by three given points.
- Verify collinearity of points.

Coordinate geometry helps us to study geometry using algebra, and understand algebra with the help of geometry. Because of this, Coordinate geometry is widely applied in various fields such as physics, engineering, navigation, seismology and art.

By the end of the lesson, children should be able to appreciate the relation between algebra and geometry.

EXPECTED LEARNING OUTCOMES

1. To understand the terms abscissa, ordinate, quadrant.
2. To understand a point on x-axis is (x,0) and a point on y-axis is (0,y)
3. To find the distance between two points in a plane using distance formula.
4. To find the type of triangle formed when the coordinates of 3 points are given.
5. To find the type of quadrilateral formed when the coordinates of 4 points are given.
6. To understand the condition for collinearity
7. To understand the section formula and its corollary- midpoint formula and its applications.
8. To understand the formula for finding area of a triangle in a plane and its applications.
CONCEPT MAP

COORDINATE GEOMETRY

STUDY THAT RELATES ALGEBRA TO GEOMETRY IN A CARTESIAN PLANE

DISTANCE FORMULA
To find the ratio in which a point divides a line segment
To find distance between 2 points on a plane

SECTION FORMULA

AREA OF TRIANGLE
To find area of triangle in a cartesian plane

MIDPOINT FORMULA
To find midpoint of a line segment

CONDITION FOR COLLINEARITY
GRADED EXERCISES

LEVEL-I

1. Find the distance of the point (4,3) from the origin. 
   (1mark)
2. Show that the points (1,1), (3,-2) and (-1, 4) are collinear. 
   (2marks)
3. Find a point on y axis which is equidistant from the points (3, 4) and (6, 7). 
   (2marks)
4. Find a point on the x-axis which is equidistant from the points A (5,4) and B(-2,3) 
   (2marks)
5. Show that (-2, 1), (2, -2) and (5, 2) are the vertices of a right triangle. 
   (2marks)
6. Prove that the points (7,10) , (-2,5) and (3,-4) are the vertices of an isosceles right triangle. 
   (2marks)
7. If the distance of P(x, y) from A (5, 1) and B (-1, 5) are equal, P.T.3x = 2y. 
   (2marks)
8. Prove that the points (7,3), (3,0), (0, -4) and (4, -1) are the vertices of a rhombus. 
   (3marks)
9. Find the coordinates of a point which divides internally the line segment joining the points (-3,-4) and (-8, 7) in the ratio 7:5. 
   (3marks)
10. Find the ratio in which the point (-2, 3) divides the line segment joining the points (-3,5) and(4,-9) 
    (3marks)
11. Find the ratio in which y axis divides the line segment joining the points (-4,5) and (3, -7). 
    (3marks)
12. Find the ratio in which the line segment joining A (1, -5) and B (-4, 5) is divided by the x- axis. Also, find the coordinates of the point of division. 
    (3marks)
13. Find the area of triangle formed by the points (4, 3), (5, 4) and (11, 2). 
    (2marks)
14. Find k so that A(-2,3), B(3, -1) and C(5, k) are collinear. 
    (2marks)
15. The coordinates of vertices of ΔABC are A(-5,-1) , B(3,-5) and C(5,k). If the area of ΔABC is 32 sq.units, find the value of k. 
    (2marks)

LEVEL-II

1. Find the coordinates of the points on the x-axis, which are at a distance of 5 units from the point (5,4) 
   (2marks)
2. Find a relation between x and y such that the point (x,y) is equidistant from the points (3,6) and (-3,4) 
   (2marks)
3. Find the value of x such that PQ = QR where coordinates of P, Q and R are (6, -1), (1,3) and ( x,8) respectively. 
   (2marks)
4. Find the length of the median CD of the triangle ABC, whose vertices are A (6,8), B (-4, 2), C (5, -1) 
   (2marks)
5. If (1,2),(4,y) ,(x,6) and (3,5) are the vertices of parallelogram taken in order, find the values of x and y. 
   (2marks)
6. The vertices of a parallelogram taken in order is (1,-2), (3, 6) and (5, 10). Find the co-ordinate of the 4th vertex. 
   (2marks)
7. If the midpoint of the line segment joining the points P (-3,1) and Q(x,y) is (1,2), find the co-ordinates of Q. 
   (2marks)
8. If the points (4,3) and (x,5) are on the circle with centre (2,3), find the value of x. 
   (2marks)
9. Show that the points (5,6), (1,5), (2.1) and (6,2) are the vertices of a square. 
   (3marks)
10. Find the coordinates of the points which trisect the line joining (1, -2) and (-3, 4). (3 marks)

11. Find the coordinates of the points which divide the line joining the points (-2, 0) and (0, 8) in four equal parts. (3 marks)

12. If the point C (-1, 2) divides the line segment joining A (2, 5) and B in the ratio 3:4, find the coordinates of B. (3 marks)

13. Find the ratio in which the point P (b, 1) divides the join of A (7, -2) and B (-5, 6). Also, find the value of ‘b’. (3 marks)

14. The coordinates of vertices of ΔABC are A(4, -6), B(3, -2) and C(5, 2). Prove that the median divides it into two triangles of equal area. (3 marks)

15. Find the area of quadrilateral, the coordinates of whose vertices are (-4, -2), (-3, -5), (3, -2) and (2, 3), taken in order. (4 marks)

**LEVEL-III**

1. Find the length of medians of a triangle having vertices A (0, -1), B (2, 1) and C(0, 3). (4 marks)

2. The length of a line segment is 10. If one end is at (2, -3) and the abscissa of the second end is 10. Show that its ordinate is either 3 or -9. (3 marks)

3. The line segment joining the points (3, -4) and (1, 2) is trisected at the points (a, -2) and Q(5/3, b), find the values of a and b. (3 marks)

4. Prove that the points (4, 5), (7, 6), (6, 3) and (3, 2) are the vertices of a parallelogram. Is it a rectangle? (4 marks)

5. The line joining the points A (4, -5) and B (4, 5) is divided by the point P such that \( \frac{AP}{AB} = \frac{2}{5} \). Find the coordinates of P. (3 marks)

6. Show that the points (-1, 1), (1, 1) and \((-\sqrt{3}, \sqrt{3})\) are the vertices of an equilateral triangle. (3 marks)

7. In what ratio does the line 2x + y = 4 divides the line segment joining A (2, -2) and B (3, 7)? Also, find the coordinates of the point of intersection. (3 marks)

8. Find the coordinates of centre of the circle passing through the points (0, 0), (-2, 1) and (-3, 2). Also, find the radius. (4 marks)

9. The points A (2, 9), B (a, 5) and C(5, 5) are the vertices of a triangle right angled at B. Find the value of ‘a’. (3 marks)

10. If P (x, y) is any point on the line segment joining the points A (a, 0) and B (0, b), show that \( \frac{x}{a} + \frac{y}{b} = 1 \). (3 marks)

**VALUE BASED QUESTIONS**

1. To raise social awareness about hazards of smoking, a school decided to start "No Smoking" campaign. 10 students are asked to prepare campaign banners in the shape of triangle as shown in the fig. If cost of 1 cm² of banner is Rs. 2, then find the overall cost incurred on such campaign. Which value is depicted in the question?
2. The coordinates of the houses of Sameer and Rahim are (7, 3) and (4, -3) whereas the coordinates of their school is (2, 2). If both leaves their houses at the same time in the morning and also reaches school on time then who travel faster? Which value is depicted in the question?

3. The students of class X of a school undertake to work for the campaign ‘Say No to Plastic’ in a city. They took the map of the city and form coordinate plane on it to divide their areas. Group A took the region covered between the coordinates(1,1), (-3,2), (-2,-2) and (1,-3) taken in order. Find the area of region covered by group A.
   (i) What are the harmful effects of using plastic?
   (ii) How can you contribute in spreading awareness for such campaign?

**ERROR ANALYSIS**

<table>
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</tr>
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<tbody>
<tr>
<td>1</td>
<td>Some students find midpoint of the line segment joining the given points instead of taking ratio as K:1</td>
<td>Concept of section formula should be explained for different situations</td>
</tr>
<tr>
<td></td>
<td>Some students get confused when ratio is given as $\frac{6}{25}$: 1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>While solving questions students exchange the ordered pairs and sometimes do not take the negative sign</td>
<td>Concept should be made clear about the coordinates and practice should be given</td>
</tr>
<tr>
<td>3</td>
<td>Condition for collinearity – confusion in use of distance formula and area of triangle formula</td>
<td>Students should be thoroughly drilled to use the appropriate formula as per the given problem.</td>
</tr>
<tr>
<td>4</td>
<td>Recognising the type of quadrilateral when coordinates of vertices are given</td>
<td>Properties of different types of quadrilaterals should be thoroughly explained to distinguish from each other</td>
</tr>
</tbody>
</table>

**QUESTION BANK**

1. If the midpoint of the line segment joining the points A(3,4) and B (k,6) is P(x,y) and $x + y – 10 = 0$, find the value of ‘k’.
2. Find the area of triangle ABC with A (1,-4) and the midpoints of sides through A being(2,-1) and (0, -1)
3. If the points A (1,-2), B (2,3), C (a,2) and D (-4,-3) form a parallelogram, find the value of ‘a’ and the height of parallelogram taking AB as base.

4. If A (6,1), B (8,2) and C (9,4) are the three vertices of a parallelogram ABCD, and E is the midpoint of DC, find the area of triangle ADE.

5. If D (-1/2,5/2), E (7,3) and F (7/2,7/2) are the midpoints of sides of triangle ABC, find the area of ΔABC.

6. Find the centre of a circle passing through the points (6,-6), (3,-7) and (3,3)

7. Two opposite vertices of a square are (-1,2) and (3,2). Find the coordinates of other two vertices.

8. If A, B, P are the points (-4,3), (0,-2) and (a,b) respectively and P is equidistant from A and B, show that 8a – 10b + 21 = 0

**ACTIVITIES**

1. To verify the distance formula using graph paper.
2. To verify the section formula using graph paper.
3. To verify the formula for area of triangle using graph paper.

**PROJECTS**

(i) To mark coordinate axes on your city map and find distances between important landmarks - bus stand, railway station, airport, hospital, school, your house etc.

(ii) Displacement and Rotation of a geometrical figure.

**POWER POINT PRESENTATIONS**

**WEB LINKS**

- COORDINATE GEOMETRY
  https://www.youtube.com/watch?v=em0gFD-bT18
- Distance formula example
  https://www.youtube.com/watch?v=QPIWrQyeuYw
- Formula
  https://www.youtube.com/watch?v=8cq7mGjLYhI
- Section formula
  https://www.youtube.com/watch?v=6KpDWu5gKww
- Area of triangle
  https://www.youtube.com/watch?v=HgzF4pAZURY
CHAPTER-9
SOME APPLICATIONS OF TRIGONOMETRY

INTRODUCTION

Concepts
1. Trigonometric Identities
2. Line of sight
3. Angle of Elevation
4. Angle of Depression

You have already introduced the branch of mathematics called Trigonometry and the concepts of trigonometric ratios of an acute angle of a right angled triangle such as $\sin \theta$, $\cos \theta$, $\tan \theta$, $\sec \theta$, $\csc \theta$, and $\cot \theta$. You have also obtained trigonometric ratios of some specific angles namely $90^0$, $60^0$, $45^0$, $30^0$, and $0^0$ geometrically. Now in this chapter we shall recall all these concepts briefly and extend this knowledge of trigonometry to establish some trigonometric identities and solve some daily life problems on Heights and distances.

Trigonometry and its applications is the extension unit of Introduction to Trigonometry. So, the teacher can start the chapter by taking lots of exercises related to the previously learnt concepts. Moreover, this unit is application of trigonometry, so lots of problem solving skills are required.

As a first warm up (W1) activity students can be motivated to speak about all the terms related to right angled triangle. Terms can be displayed on the screen or teacher can rotate a basket with paper slips in the class. Students can pick up a slip and speak about the word written over it. Students can give definition, their understanding of the term in their own words or any other information beyond the textbook knowledge. Each child should be encouraged by the teachers to speak without commenting right or wrong on his statements. If there is anything wrong in the child’s answers, teacher can initiate a discussion or can illustrate with the help of drawing/picture.

Further warm up activities like a cartoon strip with a statement in the bubble will allow the students to perceive the situation in their own way. By writing their views on it and thinking loudly they can reach to the conclusion that trigonometric ratios are independent of the length of the sides and dependent on the angles between the two sides.

Some more pre content activities can be conducted in the class to reinforce the knowledge of trigonometric ratios, value of T-ratios at prescribed angles i.e. $0^0$, $30^0$, $45^0$, $60^0$, $90^0$. These tasks will help the students in following ways:

1. To refresh the knowledge and the memory based concepts.
2. To acquire the skills in using them in problem solving.
3. To gear them for learning further concepts such as trigonometric identities and height and distance problems requiring knowledge of T-ratios and identities.

EXPECTED LEARNING OUTCOMES

1. To understand the use of studying trigonometry and the occupations in which this concept is used.
2. The basic knowledge of trigonometric ratios.
3. To know the trigonometric ratios of some specific angles.
4. To understand the concept and differentiate between line of sight, angle of elevation and angle of depression.

5. To understand that when the observer moves towards the perpendicular distance the angle of elevation increases and moves away the angle of elevation decreases.

6. To understand that the angle of elevation and angle of depression is always acute angles.

7. To draw correct and appropriate figures to the verbal sums given.

8. To solve correctly by applying trigonometric ratios in right triangles which are formed by the given information.

9. To solve verbal sums based on
   (a) Finding angle of a right triangle when two sides are given.
   (b) Finding one side of a right triangle when an acute angle and one of the other two sides are given.
   (c) Two right triangles having common base or perpendicular.
   (d) Use of two right triangles when length of one side of each triangle is equal or a relation between them is known.
   (e) Right angled triangles formed by angle of depression.

10. The skill of drawing accurate figures, skill of interpretation and skill of solving with appropriate method.
GRADED EXERCISES

LEVEL I

1. If the length of the shadow of a man is equal to the height of man. Then find the angle of elevation
   (1M)

2. If the length of the shadow of a pole 30 m high at some instant is $10 \sqrt{3}$ m, then find the angle of
   elevation of the sun (1M)

3. Find the angle of depression of a boat from the bridge at a horizontal distance of 25 m from the bridge, if the
   height of the bridge is 25 m. (1M)
4. The tops of two poles of height 10m and 18m are connected with wire. If wire makes an angle of 30° with horizontal, then find the length of wire (1M)

5. From a point 20m away from the foot of the tower, the angle of elevation of the top of the tower is 30°. Then find the height of the tower (1M)

6. The ratio of the length of a tree and its shadow is $1 : \frac{1}{\sqrt{3}}$. Then find the angle of elevation of the sun? (1M)

7. A kite is flying at a height of $50 \sqrt{3}$ m above the level ground, attached to string inclined at 60° to the horizontal, then find the length of string (1M)

8. In given fig. 2 the perimeter of rectangle $ABCD$ is (1M)

9. A tree is broken at a height of 10 m above the ground. The broken part touches the ground and makes an angle of 30° with the horizontal. Hence find the height of the tree (2M)

10. If the shadow of a tree is $\frac{\pi}{3}$ times the height of the tree, and then find the angle of elevation of the sun (1M)

11. The height of a tower is 50 m. When angle of elevation changes from 45° to 30°, the shadow of tower becomes x metres more, then find the value of x (1M)

12. The angle of elevations of a building from two points on the ground 9m and 16m away from the foot of the building are complementary, then find the height of the building (2M)

13. If the angle of elevation of a tower from a distance of 100 m from its foot is 60° then find the height of the tower (3 M)

14. A 6 feet tall man finds that the angle of elevation of a 24 feet high pillar and the angle of depression of its base are complimentary angles. Find the distance of the man from the pillar (4M)

**LEVEL II**

1. In $\triangle ABC$, $AB=5cm$, $\angle ACB=30^0$. Find AC and BC.
2. From a point 20m away from the foot of a tower, the angle of elevation of top of the tower is 30°, find the height of the tower.

3. A ladder 50m long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, find the height of the wall.

4. A circus artist is climbing a 20m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30°.

5. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8m. Find the height of the tree.

6. The shadow of a tower standing on a level plane is found to be 50m longer when sun’s elevation is 30° then when it is 60°. Find the height of the tower.

7. The angle of depression of the top and bottom of a tower as seen from the top of a 100m high cliff are 30° and 60° respectively. Find the height of the tower.

8. From a window (9m above ground) of a house in a street, the angles of elevation and depression of the top and foot of another house on the opposite side of the street are 30° and 60° respectively. Find the height of the opposite house and width of the street.

9. From the top of a hill, the angle of depression of two consecutive kilometer stones due east are found to be 30° and 45°. Find the height of the hill.

10. Two poles of equal heights are standing opposite each other on either side of the road, which is 80m wide. From a point between them on the road the angles of elevation of the top of the poles are 60° and 30°. Find the heights of pole and the distance of the point from the poles.

11. The angle of elevation of a jet fighter from a point A on the ground is 60°. After a flight of 15seconds, the angle of elevation changes to 30° if the jet is flying at a speed of 720km/ hr, find the constant height at which the jet is flying.

12. A window in a building is at a height of 10m above the ground. The angle of depression of a point P on the ground from the window is 30°. The angle of elevation of the top of the building from the point P is 60°. Find the height of the building

13. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of tower with a uniform speed. Six minutes later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower.

14. If the ratio between the length of the shadow of a tower and its height is $\sqrt{3} :1$, then what is the angle of elevation of Sun?

15. From a window 15 meters high above the ground in a street, the angles of elevation and depression of the top and foot of another house on the opposite side of the street are 30° and 45° respectively. Show that the height of the opposite house is 23.66 meters (Take $\sqrt{3} = 1.732$)
LEVEL III

1. A man standing on the deck of a ship, which is 10m above the water level, observes the angle of elevation of the top of a hill as 60° and angle of depression of the base of the hill as 30°. Find the distance of the hill from the ship and the height of the hill.

   (4M)

2. A pole of height 5m is fixed on the top of the tower. The angle of elevation of the top of the pole as observed from a point A on the ground is 60° and the angle of depression of the point A from the top of the tower is 45°. Find the height of tower. (Take √3 = 1.732)

3. From a point on the ground the angle of elevations of the bottom and top of a water tank kept on the top of the 30m high building are 45° and 60° respectively. Find the height of the water tank.

4. The shadow of a tower standing on the level ground is found to be 60m shorter when the sun’s altitude changes from 30° to 60°, find the height of tower.

5. The angle of elevation of a bird from a point on the ground is 60°, after 50 seconds flight the angle of elevation changes to 30°. If the bird is flying at the height of 500√3 m. find the speed of the bird.

6. The angle of elevation of a jet fighter plane from a point A on the ground is 60°. After a flight of 15 seconds, the angle of elevation changes to 30°. If the jet is flying at a speed of 720 km/h. find the constant height at which the jet is flying.

7. From a window 20m high above the ground in a street, the angle of elevation and depression of the top and the foot of another house opposite side of the street are 60° and 45° respectively. Find the height of opposite house.

8. An aeroplane flying at a height of 1800m observes angles of depressions of two points on the opposite bank of the river to be 60° and 45°, find the width of the river.

9. The angle of elevation of the top of the tower from two points A and B which are 15m apart, on the same side of the tower on the level ground are 30° and 60° respectively. Find the height of the tower and distance of point B from the base of the tower.

10. The angle of elevation of the top of a 10m high building from a point P on the ground is 30°. A flag is hoisted at the top of the building and the angle of elevation of the top of the flag staff from P is 45°. Find the length of the flag staff and the distance of the building from point P.

11. The angle of elevation of a bird from a point 12 metres above a lake is 30° and the angle of depression of its reflection in the lake is 60°. Find the distance of the bird from the point of observation.

12. The shadow of a vertical tower on level ground increases by10 mtrs. When sun’s attitude changes from 45° to 30°. Find the height of the tower, up to one place of decimal.

13. A man on a cliff observes a boat at an angle of depression of 30°, which is approaching the shore to point ‘A’ immediately beneath the observer with a uniform speed, 12 minutes later, the angle of depression of the boat is found to be 60°. Find the time taken by the boat to reach the shore.
14. A man standing on the deck of a ship, 18m above the water level observes that the angle of elevation and depression of the top and the bottom of a cliff are 60° and 30° respectively. Find the distance of the cliff from the ship and height of the cliff.

15. A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is 60°. When he moves 40m away from the bank he finds the angle of elevation to be 30°. Find the height of the tree and the width of the river.

16. An aeroplane, when 300 m high, passes vertically above another plane at an instant when the angle of elevation of two aeroplanes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the two planes.

17. The angle of depression of the top and bottom of a 10m tall building from the top of a tower are 30° and 45° respectively. Find the height of the tower and distance between building and tower.

18. A boy standing on a horizontal plane finds a bird flying at a distance of 100m from him at an elevation of 30°. A girl, standing on the root of 20m high building, finds the angle of elevation of the same bird to be 45°. Both the boy and girl are on the opposite sides of the bird. Find the distance of bird from the girl.

19. A man standing on the deck of a ship, which is 10m above the water level observes the angle of elevation of the top of the hill as 60° and the angle of depression of the base of the hill is 30°. Calculate the distance of the hill from the ship and the height of the hill.

20. The angle of elevation of a building from two points P and Q on the level ground on the same side of the building are 36° and 54° respectively. If the distance of the points P and Q from the base of the building are 10m and 20m respectively, find the height of the building.

VALUE BASED QUESTIONS

1. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. The teacher asked the students to find the height of the tree. All the students failed but Neeraj took initiative and calculated it correctly using trigonometry.
   (i) What height Neeraj calculated?
   (ii) Which mathematical concept is used to solve the question?
   (iii) What quality of Neeraj is depicted here?

2. A person, standing on the bank of a river, observes that the angle subtended by a tree on the opposite bank is 60°. When he retreats 20 m from the bank, he finds the angle to be 30°.
   (i) Find the height of the tree and the breadth of the river?
   (ii) What skill is used by the person?

3. Anand is watching a circus artist climbing a 20 m long rope which is tightly stretched and tied from the top of a vertical pole to the ground.
(i) Find the height of the pole if the angle made by the rope with the ground level is 30°.
(ii) What value is experienced by Anand?

1. A pilot is flying an aeroplane at an altitude of 1800 m observes that two ships are sailing towards it in the opposite directions. The angles of depressions of the ships as observed from the aeroplane are 60° & 30° respectively.
   (i) Find the distance between the two ships?
   (ii) What value of the pilot is shown?

2. The angle of elevation of a bird observed by a hunter who is 12 m above a lake is 30° and the angle of depression of bird’s reflection in the lake is 60°.
   (i) Find the distance between the bird and the hunter.
   (ii) What value is used by the hunter if he wants to hit the bird?

3. A boy sees an injured cat on a window sill 20 m above the ground. To help the cat the boy takes the staircase at an angle of elevation of 45°
   (i) What is the distance the boy has to cover to reach the cat?
   (ii) What are the qualities the boy projects to help the cat?

4. A Para-glider when gliding 1000 m above a forest notices a forest fire. He also observes two fire stations at angles of 45° and 60° on either side of the fire.
   (i) Find the distance of each fire station from the point of fire.
   (ii) Which station should the para-glider intimate first about the forest fire and what value do you learn from him?

5. A builder is asked to build a decorative pillar 100 m high. After the completion of the construction a surveyor came and viewed the top of the pillar from a point 55 m away from the foot of the pillar at an angle of elevation of 30°
   (i) What is the actual height of the pillar? (Take √3 = 1.732)
   (ii) Do you think that the builder must be paid fully? Justify your answer.

6. An air traffic controller instructs a plane on the edge of a runway to take off at 45°. At the same time he instructs a plane flying at a height of 1 km to descend at 30° to reach the edge so that both do not collide
   (i) Find the difference in the heights of the planes at the instant when one plane is vertically above the other
(ii) What are the values you learn from the air traffic controller?

7. A lighthouse 100m high emits light such that the farthest reach of the light to the ship on the sea was an angle of 45°. But it was noticed that many ships ran ashore and were wrecked.
   (i) What should be the height of the lighthouse so that the reach of the light would be at an angle of 30°?
   (ii) What values do you think the authorities must possess to rectify the problem?

8. The captain of ship A sees a distress flare fired by a ship in distress at an angle of elevation of 60°. At the same instant the captain of ship B also sees the same flare at angle of elevation of 45°. The flare is approximately 1000m above sea level.
   (i) Find the distance of both the ships from the ship in distress
   (ii) What qualities must the captains of both the ships possess to help the ship in distress?

9. Ram is standing on the window of the first floor of a building observes a person throwing a garbage into the dustbin which is 10m from the foot of the building with angle of depression of 45°. He climbs to the window of second floor directly above the first floor and observes the same activity of the person with the angle of depression of the dustbin to be 60°.
   (i) Find the height of the first and second floor.
   (ii) What values should a person inculcate to keep the environment clean?

10. A flag staff stands on the top of 5m high tower. From a point on the ground the angle of elevation of the top of the flag staff is 60° and from the same point the angle of elevation of the top of the tower is 45°.
    (i) Find the height of the flag staff.
    (ii) Mention any two ways in which every citizen of India should respect the national flag.

11. A person standing on the bank of a river observes that the angle of elevation of the top of a building of an organization working for conservation of wildlife, standing on the opposite bank is 60°. When he moves 40m away from the bank, he finds the angle of elevation to be 30°.
    (i) Find the height of a building and width of the river.
    (ii) Why did we need to conserve wild life?

12. Suppose there are two windows in a house. A window of the house is at a height of 1.5 m above the ground and the other window is 3m vertically above the lower window. Anil and Sanjeev are sitting inside the two windows. At an instant, the angles of elevation of a balloon from these windows are observed as 45° and 30°, respectively.
    (i) Find the height of the balloon from the ground.
    (ii) Among Anil and Sanjeev, who is closer to the balloon?
(iii) Which values are added by windows in any construction commercial or residential?

(iv) If the balloon is moving towards the building, then will the angles of elevation remain the same?

13. The contractor was awarded to construct a vertical pillar at a horizontal distance of 100m from a fixed point. It was decided that angle of elevation of the top of the complete pillar from that point was to be \(60^\circ\). Contractor finished the job by making a pillar such that the angle of elevation of its top was \(45^\circ\).

(i) Find the height of the pillar to be increased as per the terms of contract.

(ii) Contractor demands full payment for this work.

(a) Is he justified

(b) Which value he is lacking?

14. A man on a cliff observes a boat at an angle of depression of \(30^\circ\). Which is approaching the shore with uniform speed. After 6 minutes it was observed at an angle of depression \(60^\circ\)?

a) Find the time taken by the boat to reach the shore.

b) Which mathematical concept is used in the above problem?

c) What is its value?

ERROR ANALYSIS

<table>
<thead>
<tr>
<th>Sl no</th>
<th>POSSIBLE ERRORS</th>
<th>REMEDIATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Could not make correct figures for word problems so no solution after that</td>
<td>Stress must be made to draw correct figure after reading the statement</td>
</tr>
<tr>
<td>2</td>
<td>Many students could not translate the word problems to a correct diagram</td>
<td>Sufficient practice of making correct diagram Its importance should be emphasized</td>
</tr>
<tr>
<td>3</td>
<td>A few students take (\tan 60^\circ = \frac{1}{\sqrt{3}}) and (\tan 30^\circ = \sqrt{3}).</td>
<td>Values of trigonometric ratios e.g. (\tan 60^\circ), (\tan 30^\circ) etc. should be correctly used, and so sufficient practice should be given.</td>
</tr>
<tr>
<td>4</td>
<td>Given value of (\sqrt{3} = 1.732) was not used in word problems</td>
<td>Value of (\sqrt{3}) if given in the question, has to be used as such</td>
</tr>
<tr>
<td>5</td>
<td>Some students draw untidy construction</td>
<td>Emphasis should be given on proper labeling of figures and neat diagrams</td>
</tr>
<tr>
<td>6</td>
<td>Some students do not do the calculation correctly and do not write the units in the answer</td>
<td>Importance of writing units and accurate calculation should be emphasized</td>
</tr>
</tbody>
</table>

QUESTION BANK

1. A ladder 15 m long lean against a wall making an angle of \(60^\circ\) with the wall. Find the height of the point where the ladder touches the wall.

2. If the angles of elevation of the top of a tower from two points distance \(a\) and \(b\) (\(a>b\)) from its foot and in the same straight line from it are respectively \(30^\circ\) and \(60^\circ\), then find the height of the tower.

3. Angles of depression from the top of a light - house of two boats are \(45^\circ\) and \(30^\circ\), which are 60 m apart due east. Find the height of light house (in m).
4. A ladder makes an angle of $60^\circ$ with the ground when placed against a wall. If the foot of ladder is 2 m away from the wall. Find the length of ladder.
5. The angle of depression of a car parked on the road from the top of a 150 m high tower is $30^\circ$. Find the distance of the car from the tower (in m).
6. If the length of the shadow of an object is greater than the height of the object, then what is the angle of elevation?
7. The angle of depression of a car, standing on the ground, from the top of a 75 m high tower, is $30^\circ$. Find the distance of the car from the base of the tower (in m).
8. An observer 1.5 m tall is 28.5 m away from a tower 30 m high. Find the angle of elevation of the top of the tower from his eye.
9. The height of the tower is 100 m, when the angle of elevation of sun is $30^\circ$, then what will be the length of shadow of the tower?
10. The top of two poles of height 16 m and 10 m are connected by a wire of length L metres. If the wire makes an angle of $30^\circ$ with the horizontal, then find L.
11. A kite is flying at a height of 90 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is $60^\circ$. Find the length of the string assuming that there is no slack in the string.
12. From a point P on the ground the angle of elevation of the top of a 10 m tall building is $30^\circ$. A flag is hoisted at the top of the building and the angle of elevation of the top of the flagstaff from P is $45^\circ$. Find the length of the flagstaff and distance of building from point P.
13. A tower stands vertically on the ground. From a point on the ground which is 60 m away from the foot of the tower, the angle of elevation of the top of the tower is found to be $60^\circ$. Find the height of the tower.
14. If the shadow of a tower is 30 m long, when the sun’s elevation is $30^\circ$. What is the length of the shadow, when sun’s elevation is $60^\circ$?
15. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle $30^\circ$ with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.
16. A circus artist is climbing from the ground along a rope stretched from the top of a vertical pole and tied at the ground. The height of the pole is 12 m and the angle made by the rope with ground level is $30^\circ$. Calculate the distance covered by the artist in climbing to the top of the pole.

**ACTIVITIES**

**MAKING A CLINOMETER**

**Objective**

To make a clinometer and use it to measure the height of an object.

**Pre-requisite knowledge**

1. Properties of right angled triangles

**Materials required**

Stiff card, small pipe or drinking straw, thread, a weight (a metal washer is ideal)

**Procedure:**

1. Prepare a semi-circular protractor using any hard board and fix a viewing tube (Straw or pipe) along the diameter.
2. Punch a hole ($o$) at the center of the semicircle.
3. Suspend a weight \( w \) from a small nail fixed to the center.
4. Ensure that the weight at the end of the string hangs below the protractor.
5. Mark degrees (in hexadecimal scale with 00 at the lowest and 10 to 900 proceeding both clockwise and anticlockwise). [Fig 21].

Determining the height of an object
6. First measure the distance of the object from you. Let the distance be \( d \).
7. Look through the straw or pipe at the top of the object. Make sure you can clearly see the top of the object.
8. Hold the clinometer steady and let your partner record the angle the string makes on the scale of the clinometer. Let this angle be \( \Theta \).

Using trigonometric ratio:

\[
\tan \Theta = \frac{\text{height}}{\text{distance}} = \frac{h}{d} \\
h = d \times \tan \Theta
\]

If, for example, \( d = 100 \) m and \( \Theta = 45^\circ \),
\[
h = 100 \times \tan 45^\circ = 100 \text{ m}
\]

**Learning outcome**

Students learn how to determine the angle of elevation of an object and use it to determine the height of an object at a known distance.

**Remark**

Students may be asked to change the distance of the object (by either moving the object or by changing their position) and note how the angle of elevation varies. They will notice that though \( h \) and \( \Theta \) will vary, the product \( h = d \tan \Theta \) will be constant (within measurement error).

![Clinometer](image)

**Activity**

To recall the sides and angles in a right triangle

**Description:**
In earlier classes the students have gained the knowledge of right triangle, its sides and angles. This is a starter activity through which the students will revise the learnt concept of finding the angles and sides of a right triangle using Pythagoras theorem and angle sum property of a triangle

Execution:

Prepare some flash cards on which various right triangles are drawn. Some of the sides and angles are missing. Ask the students to find them.

Parameters for assessment:

- Able to use Pythagoras theorem
- Able to apply the angle sum property

Activity

To review and recall the knowledge of relation between T ratios at a given angle and T ratios for its complementary angle.

Description:

P2 is designed for assessing the previous knowledge of students in terms of relation between T ratios and knowledge of complementary angles.

Execution:

Ask the students to recall T ratios and do the matching exercise given in P2.

Parameters for assessment:

- Able to tell the relation between T – ratios
- Able to tell T ratios of complementary angles

Activity

To learn to label angle of elevation, angle of depression and line of sight in a given situation.

Description:

After learning to solve a right triangle using trigonometry, students will be provided an opportunity to explore its utility in daily life. For this they will first understand some basic terms like angle of depression, angle of elevation and line of sight.

Execution:

On a worksheet, various situations will be depicted through pictures. Students will mark the angle of elevation, angle of depression and line of sight.

Parameters for assessment:

- Understanding on concept
- Correct marking of angle of depression/elevation and line of sight on the worksheet

Activity
To learn to apply the knowledge of trigonometry in solving daily life problems

Description:
After learning to represent a given situation in a right triangle, have a discussion on application of trigonometry. Students will be then asked to solve the right triangle and find the unknown side using trigonometric ratios.

Execution:
Distribute the worksheet. Explain the template of how to write and solve a problem. All the students will solve the problems followed by a discussion on using trigonometry in solving problems.

Parameters for assessment:
• Drawing correct figure for a given problem
• Solving correctly and writing final answer with proper units

PROJECTS

GEOMETRY IN REAL LIFE

Description
In this project we try to find situations in daily life where geometrical notions can be effectively used. In particular, in the following examples the student discovers situations in which properties of similar triangles learnt in the classroom are useful.

1. How tall a mirror should you buy if you want to be able to see your full vertical image? We are given the fact that the angle of incidence equals the angle of reflection. Students will find that the mirror should be at least half his/her height.

2. To find the width of a pathway:
Fix a pole at Q directly opposite to a tree P on the other side of pathway

Walk along the pathway, fix another pole at R at a known distance. Walk another known distance to S. From here, walk at right angles to the pathway till the point T is reached, such that T is directly in line with R and P. Measure the distance ST. Using the property of similarity of triangles, the width of the pathway is determined.

3. To find the height of a tree:
Place a ruler upright in the shadow of the tree, so that the end of its shadow is at the same place as the end of the shadow of the tree. Knowing the relevant distances, the height of the tree can be estimated.
As part of this project students should think of examples involving different geometrical properties of triangles and circles.

Projects

1. To find the height of the tree
2. To find the width of a pathway
3. How tall a mirror should you buy if you want to be able to see your full vertical image?
   
   **Activity** –

   To recall the sides and angles in a right triangle

4. **Description:**

   In earlier classes the students have gained the knowledge of right triangle, its sides and angles. This is a starter activity through which the students will revise the learnt concept of finding the angles and sides of a right triangle using Pythagoras theorem and angle sum property of a triangle

5. **Execution:**

   Prepare some flash cards on which various right triangles are drawn. Some of the sides and angles are missing. Ask the students to find them.

6. **Parameters for assessment:**

   Able to use Pythagoras theorem
   Able to apply the angle sum property

7. **Activity**

   To review and recall the knowledge of relation between T ratios at a given angle and T ratios for its complementary angle.
8. **Description:**

P2 is designed for assessing the previous knowledge of students in terms of relation between T ratios and knowledge of complementary angles.

9. **Execution:**

Ask the students to recall T ratios and do the matching exercise given in P2.

10. **Parameters for assessment:**

   - Able to tell the relation between T –ratios
   - Able to tell T ratios of complementary angles

11. **Activity**

To learn to label angle of elevation, angle of depression and line of sight in a given Situation

12. **Description:**

After learning to solve a right triangle using trigonometry, students will be provided an opportunity to explore its utility in daily life. For this they will first understand some basic terms like angle of depression, angle of elevation and line of sight.

13. **Execution:**

On a worksheet, various situations will be depicted through pictures. Students will mark the angle of elevation, angle of depression and line of sight.

14. **Parameters for assessment:**

   - Understanding on concept
   - Correct marking of angle of depression/elevation and line of sight on the worksheet

**Activity**

To learn to apply the knowledge of trigonometry in solving daily life problems

1. **Description:**

After learning to represent a given situation in a right triangle, have a discussion on application of trigonometry. Students will be then asked to solve the right triangle and find the unknown side using trigonometric ratios.

2. **Execution:**

   Distribute the worksheet. Explain the template of how to write and solve a problem. All the students will solve the problems followed by a discussion on using trigonometry in solving problems.

3. **Parameters for assessment:**

   - Drawing correct figure for a given problem
   - Solving correctly and writing final answer with proper units
POWER POINT PRESENTATIONS

WEB LINKS

- Concepts of angle of elevation and angle of depression
  - https://www.youtube.com/watch?v=0Cz2DJ_bujo
- Tricks to remember trigonometric ratios of standard angles
  - https://www.youtube.com/watch?v=xXGfp9PKdXM
  - http://www.mathsisfun.com/right_angle_triangle.html
- Pythagorean Theorem http://www.grc.nasa.gov/WWW/K-12/airplane/pythag.html
- Sums related to heights and distances
  - https://www.youtube.com/watch?v=0Cz2DJ_bujo&list=PLA8908406020E864D
  - https://www.youtube.com/watch?v=cNq6Utj9D0&list=PLA8908406020E864D&index=3
  - https://www.youtube.com/watch?v=ZPoFq0SSxy4&index=4&list=PLA8908406020E864D
  - https://www.youtube.com/watch?v=GyGKE8JFqrg&list=PLA8908406020E864D&index=5
  - https://www.youtube.com/watch?v=UJuqo8RmxbY&index=6&list=PLA8908406020E864D
  - https://www.youtube.com/watch?v=8gTCEvlNo-0&list=PLA8908406020E864D&index=7
  - https://www.youtube.com/watch?v=W3C9unnSrRo
- http://mathcentral.uregina.ca/QQ/database/QQ.09.03/anjum1.html
- Watch Video
  - Html
- A PPT
  - http://www.slideshare.net/harshmahajan5477/maths-ppt-on-some-applications-of-trignometry
  - http://www.authorstream.com/Presentation/mhappy2day-1277073-applications-of-trigonometry/
- Online Test
- Value based questions
CHAPTER – 10

CIRCLES

INTRODUCTION

BASIC CONCEPTS

1. Point of contact
2. Tangent
3. Secant
4. Chord
5. Perpendicular bisector
6. Length of tangent

In Class IX, we have discussed a circle and basic terms such as centre, radius, arc, chord etc, related to a circle and some important properties. In this unit, we shall first briefly recall these basic concepts and properties and then extend this knowledge to learn the concept of a tangent to a circle and some properties of the tangent.

EXPECTED LEARNING OUTCOMES

1. To recognize and define a circle
2. To identify the different elements of a circle and also show by illustrations
3. Differentiate between sector and segment of a circle
4. To understand the meaning and difference between a secant and a tangent
5. To understand and illustrate pictorially the properties of a circle and use them appropriately while solving sums
   (a) The perpendicular from the centre of a circle to a chord bisects the chord
   (b) Angle in a semicircle is a right angle
   (c) Angle subtended by an arc at the centre is double the angle subtended at any other part of a circle
   (d) Angles in the same segment are equal
   (e) If two circles intersect at two points, then the line through the centres is the perpendicular bisector of the common chord
   (f) Equal chords of a circle are equidistant from the centre
6. To understand, draw correct figures and prove the theorems related to tangents stepwise and with appropriate reasoning
   (a) A tangent at any point on a circle is perpendicular to the radius at the point of contact.
   (b) The length of the tangents drawn from an external point to a circle are equal.
7. To solve problems based on
   (a) Finding length of the tangent
   (b) Finding length of the tangent from an external point to a circle
(c) To use appropriate theorems or properties
8. To draw neat and correct figures for the given verbal sums
9. To improve the skill of drawing, reasoning and select appropriate method for solving
GRADED EXCERCISES

LEVEL I

1. In the given figure if PQ is tangent then find the value of \( \angle POQ + \angle QPO \) (1M)

Ans:- 90°

2. In the adjoining figure, \( \triangle ABC \) is circumscribing a circle, then find the length of BC (1M)

Ans: - 9 cm

3. From a point P, two tangents PA and PB are drawn to a circle C (O, r). If OP =2r, then name the type of \( \triangle APB \).(1M)

Ans. Equilateral Triangle

4. In a \( \triangle ABC \), \( AB = 8 \text{cm} \), \( \angle ABC = 90^\circ \). Then find the radius of the circle inscribed in the triangle. (1M)

Ans:- 2 cm

5. If PA and PB are two tangents drawn to a circle with centre O, from an external point P such that PA=5cm and \( \angle APB = 60^\circ \), then find the length of the chord AB. (2M)

Ans:- 5 cm

6. AB is a chord of length 9.6cm of a circle with centre O and radius 6cm. If the tangents at A and B intersect at point P then find the length PA. (3M)

Ans:-8 cm

7. Prove that the lengths of tangents drawn from an external point to a circle are equal. Hence, find BC, if a circle is inscribed in \( \triangle ABC \) touching AB, BC & CA at P, Q & R respectively, having AB=10 cm, AR=7 cm & RC=5 cm. (4M)

Ans:-8 cm

8. The tangent at a point C of a circle and diameter AB when extended intersect at P. If \( \angle PCA = 110^\circ \), find \( \angle CBA \). (4M)

Ans:- 70°
9. In figure, if from an external point T, TP and TQ are two tangents to a circle with centre O so that \( \angle POQ = 110^\circ \) then find \( \angle PTQ \).

Ans: -70\(^\circ\)

10. To draw a pair of tangents to a circle which is inclined to each other at an angle of 45\(^\circ \), it is required to draw tangents at the end points of those two radii of the circle. Then find the angle between the two radii.

Ans: -135\(^\circ\)

11. What is the length of the tangent drawn from a point 8 cm away from the centre of a circle, of radius 6 cm?

Ans: \(4 \sqrt{7}\)

12. Two parallel lines touch the circle at points A and B separately. If the area of the circle is 25\(\pi\) cm\(^2\), then find the length of AB.

Ans: -10 cm

13. In the figure, O is the centre of the circle and AB is a tangent at A. If OA = 3 cm and AB = 4 cm, then calculate the length of BO.

(1M)

14. Find the distance between two parallel tangents of a circle of radius 3 cm.

Ans: -5 cm

15. A circle is inscribed in a \(\triangle ABC\) having sides AB = 8 cm, BC = 10 cm, and CA = 12 cm as shown in the figure. Find AD, BE, and CF.

Ans: AD = 5 cm, BE = 3 cm, and CF = 1 cm
LEVEL II

1. Prove that a parallelogram circumscribing a circle is a rhombus.  (3M)
2. If radii of the two concentric circles are 15 cm and 17 cm, then find the length of each chord of one circle which is tangent to one other.  (1M)
   Ans: - 16 cm
3. PQ is a chord of a circle and R is point on the minor arc. If PT is a tangent at point P such that \( \angle QPT = 60^\circ \) then find \( \angle PRQ \).  (1M)
   Ans: - 120°
4. If a tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q such that \( OQ = 12 \) cm then find the length of PQ.  (1M)
   Ans: - 119
5. If the angle between two radii of a circle is 130°, then find the angle between the tangents at the end of the radii.  (1M)
   Ans: - 50°
6. ABCD is a quadrilateral. A circle centered at O is inscribed in the quadrilateral. If \( AB = 7 \) cm, \( BC = 4 \) cm, \( CD = 5 \) cm then find DA.  (1M)
   Ans: - 8 cm
7. Two tangents PA and PB are drawn from an external point P to a circle with centre O. Prove that OAPB is a cyclic quadrilateral  (2M)
8. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle  (2M)
9. If quadrilateral ABCD is drawn to circumscribe a circle then prove that \( AB + CD = AD + BC \).  (3M)
10. PQ and PR are two tangents drawn to a circle with centre O from an external point P. Prove that \( \angle QPR = 2 \angle OQR \).  (3M)
11. A circle touches the side BC of a triangle ABC at a point P and touches AB and AC when produced, at Q & R respectively. Show that \( AQ = 1/2 \) (perimeter of \( \triangle ABC \)).  (4M)
12. If AB is a chord of a circle with centre O, AOC is diameter and AT is the tangent at the point A, then prove that \( \angle BAT = \angle ACB \).  (3M)
13. Two concentric circles are of radii 5 cm and 3 cm and centre at O. Two tangents PA and PB are drawn to two circles from an external point P such that \( AP = 12 \) cm (see figure). Find length of BP.  (2M)
   Ans: - \( 4 \sqrt{10} \)
14. In fig. two circles with centre A and B touch each externally. PM=15 cm is tangent to circle with centre A and QN=12 cm is tangent to circle with centre B from external points P & Q. If PA=17 cm and BQ=13 cm. Find the distance between the centres A and B of circles.
   Ans: - (13 cm)

15. Find the distance between the centres A and B of circles.

   \[ \text{Ans: } 13 \text{ cm} \]

16. Find \( \angle CAD \) in the following figure

   \[ \text{Ans: } 60^0 \]

**Level III**

1. If two tangents making an angle of 120\(^0\) with each other are drawn to a circle of radius 6 cm, then find the angle between the two radii, which are drawn to the tangents.
   \[ \text{Ans: } 60^0 \]

2. CP and CQ are tangents from an external point C to a circle with centre O. AB is another tangent which touches the circle at R and intersects PC and QC at A and B respectively. If CP = 11 cm and BR = 4 cm, then find the length of BC.
   \[ \text{Ans: } 7 \text{ cm} \]

3. If all the sides of a parallelogram touch a circle, show that the parallelogram is a rhombus.

4. In adjacent figure; AB & CD are common tangents to two circles of unequal radii. Prove that AB=CD.

5. Prove that the angle between the two tangents to a circle drawn from an external point is supplementary to the angle subtended by the line segment joining the points of contact to the centre.
   \[ \text{Ans: } \]  

6. The in-circle of a \( \triangle ABC \) touches the sides BC, CA & AB at D, E and F respectively. If AB=AC, prove that BD=CD.
   \[ \text{Ans: } \]  

7. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre of the circle.
8. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact. Using the above, do the following: If O is the centre of two concentric circles, AB is a chord of the larger circle touching the smaller circle at C, then prove that AC=BC. (4M)

9. From an external point P, a tangent PT and a line segment PAB is drawn to circle with centre O, ON is perpendicular to the chord AB. Prove that PA.PB=PN²-AN². (4M)

10. P In figure, XY and X’ Y’ are two parallel tangents to a circle with centre O and another tangent AB, with point of contact C intersects XY at A and X’ Y’ at B. Prove that ∠AOB =90°

11. In the figure, the radius of in circle of □ABC is 4 cm and segments into which one side BC is divided by the point of contact D are 6 cm and 8 cm. Find AB and AC

Ans: - AC= 13 cm and AB = 15 cm

12. If PA and PB are tangents from an external point P to the circle with centre O, then find ∠AOP+∠OPA. (HOTS)

Ans:- 90°

13. ABC is an isosceles triangle with AB=AC, circumscribed about a circle. Prove that the base is bisected by the point of contact.(HOTS)

14. AB is diameter of a circle with centre O. If PA is tangent from an external point P to the circle with ∠POB=115° then find ∠OPA. (HOTS)

Ans. 25°

15. PQ and PR are tangents from an external point P to a circle with centre. If ∠RPQ=120°, Prove that OP=2PQ. If the common tangents AB and CD to two circles C (O,r) and C’(O’r’) intersect at E, then prove that AB=CD.(HOTS)
VALUE BASED QUESTIONS

1. There are 3 villages A, B and C such that the distance from A to B is 7 km, from B to C is 5 km and from C to A is 8 km. The gram pradhan wants to dig a well in such a way that the distance from each village is equal.
   (i) What should be the location of well?
   (ii) Which value is depicted by gram pradhan?

2. A person of village wants to construct a road nearest to a circular village Rampur. The road cannot pass through the village. But the people wants that road should be at the shortest distance from the center of the village
   (i) Which road will be the nearest to the center of village?
   (ii) Which value is depicted by the people of village?

3. Four roads have to be constructed by touching village Khanpur in circular shape of radius 1700 m in the following manner. Savita got contract to construct the roads AB and CD while Vijay got contract to construct AD and BC.
   Prove that AB + CD = AD + BC.
   Which value is depicted by the contractor?

4. Two roads starting from P are touching a circular path at A and B. Sarita ran from P to A 10 km and Ramesh ran from P to B.
   (i) If Sarita wins the race than how much distance Ramesh ran?
   (ii) Which value is depicted?

5. A farmer wants to divide sugarcane of 7 feet length between his son and daughter equally. Divide it geometrically, considering sugarcane as a line of 7 cm, using construction.
   (i) Find the length of each part.
   (ii) Which value is depicted?
6. A group of students of the Heritage Club of a college took up the work of restoring an old circular shaped monument in their neighborhood. To help them in their work a rope of 10m length was tied from the edge of the circular dome to a pole tangentially which is 26 m from the centre of the dome
   (i) What is the radius of the circular dome?
   (ii) What values are imbibed by the students in this initiative

7. Raj tied a stone to the end of a string 80cm long and swirled it in a circular motion by holding the other end. At one instance it slipped out of his hand and the stone flew tangentially and hit the glass window pane of the neighborhood house at a distance of 1.7m from him.
   (i) Through what distance did the stone move before hitting the window?
   (ii) As a responsible student would you encourage playing such games? Justify.

8. For the inauguration of the Eco-Club of a school, Badges were given to volunteers. Sudha made these badges in the shape of a triangle with a circle inscribed in it (As given in the figure). A message supporting tree plantation was written in the circle. The length of side BC is 14 cm.
   (i) Calculate the length of sides AB and AC
   (ii) What values are imbibed by having such clubs in schools?

9. For a Science Exhibition Rahul presented a Diagrammatic representation of Rain water harvesting as a Project. AB and AC are the pipes of 5m long are bringing water from the terrace of a building.
10. (as in the Figure) The triangular space is developed as a garden.
    (i) What is the perimeter of the triangular garden?
    (ii) If the radius of the circle is 12 cm, then find the length of OA.
    (iii) What qualities do you think is encouraged by such exhibitions?

11. Mr. Sharma constructed a circular tank to serve as a bird bath. He also made fencing in the shape of a quadrilateral, the sides touching the circular tank.
    (i) If 3 sides are 6m, 7m and 4m find the length of the 4th side.
    (ii) What values does Mr. Sharma depict through his action?
12. As a part of a campaign a huge balloon with message of awareness of cancer was displayed from the terrace of a tall building. It was held by strings of length 9m each and inclined at an angle of 60° at the point where it was tied as shown in the figure.

(i) What is the length AB?

(ii) What do you think of such campaign? Should youngsters be involved more in such campaigns?

![Diagram of a balloon](image)

13. It was required to plant trees along the sides of a quadrilateral touching a circle as shown in the figure. Distance between consecutive trees was to be kept same throughout. Two gardeners Ram and Shyam were appointed by the contractor to complete the job. Ram was allotted the sides PS and QR and shyam, the other two sides. When the job was completed Ram was paid 10% more than what was paid to Shyam.

(i) Who planted more trees, Ram or Shyam

(ii) Comment on the dealings of the contractor

![Diagram of a quadrilateral](image)

**ERROR ANALYSIS**

<table>
<thead>
<tr>
<th>SNo</th>
<th>POSSIBLE ERRORS</th>
<th>REMEDIATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Given, To prove, Construction part not written by some of the students</td>
<td>Steps of proof with reasoning in order must be given enough practice</td>
</tr>
<tr>
<td>2</td>
<td>Some students draw untidy construction</td>
<td>Emphasis should be given on proper labeling of figures and neat diagrams</td>
</tr>
<tr>
<td>3</td>
<td>Some students cannot recall the result of geometry to solve problems</td>
<td>Application of the results of geometry must be given sufficient practice</td>
</tr>
<tr>
<td>4</td>
<td>Many students do not apply the result that the tangent to a circle is perpendicular to the radius at the point of contact</td>
<td>Sufficient practice by taking different problems should be given to apply the property of tangents</td>
</tr>
<tr>
<td>5</td>
<td>Some students are not able to recognize</td>
<td>Practice of writing corresponding parts of congruent</td>
</tr>
</tbody>
</table>

...
<table>
<thead>
<tr>
<th>corresponding parts of congruent triangles hence find difficult to find the values of angles</th>
<th>triangles are equal has to be given</th>
</tr>
</thead>
<tbody>
<tr>
<td>6. Some students are unable to find the fourth angle when three angles are given</td>
<td>Use of angle sum property of triangle and quadrilateral should be made clear by using ample examples.</td>
</tr>
</tbody>
</table>

**QUESTION BANK**

1. Name the line segment, having its end point on a circle.
2. Write the number of tangents that can be drawn through a point which is inside the circle.
3. Name the line that passes through point of contact and through centre of circle tangent.
4. Find the radius of a circle which is inscribed in a triangle with sides 3, 4 and 5 cm.
5. If the Distance between two parallel lines is 10 cm. Then find the radius of circle which will touch both two lines.
6. CP and CQ are tangents to a circle with centre O. ARB is another tangent touching the circle at R. If CP = 12 cm, and BC = 8cm, then find the length of BR.
7. AB is a chord of the circle and AOC is its diameter such that \(\angle ABC = 50^\circ\). If AT is the tangent to the circle at the point A, find \(\angle BAT\).
8. How many tangents can be drawn to a circle, from an external point?
9. Differentiate a secant from a tangent.
10. Name the common point between a tangent to a circle and the radius at the point at which the tangent touches it.
11. Find the length of the tangent drawn to a circle of radius 5cm with centre O, from an external point which is at a distance of 13 cm from the centre.
12. If two tangents make an angle of 120\(^\circ\) with each other, are drawn to a circle of radius 6cm, then find the angle between the two radii, which are drawn to the tangents.
13. Prove that the angle between the two tangents to a circle drawn from an external point, is supplementary to the angle subtended by the line segment joining the points of contact to the centre.
14. If quadrilateral ABCD is drawn to circumscribe a circle then prove that \(AB+CD=AD+BC\).
15. If a tangent PQ at a point P of a circle of radius 5cm meets a line through the centre O at a point Q such that \(OQ = 12\) cm then find the length of PQ.
16. Two tangents PA and PB are drawn from an external point P to a circle with centre O. Prove that OAPB is a cyclic quadrilateral.
17. If the angle between two radii of a circle is 130\(^\circ\), then find the angle between the tangents at the end of the radii.
18. Prove that the parallelogram circumscribing a circle is rhombus.
19. A circle touches the sides of a quadrilateral ABCD at P, Q, R and S respectively. Show that the angles subtended at the centre by a pair of opposite sides are Supplementary.
20. If PA and PB are tangents to a circle from an outside point P touching the circle at A and B, such that \(PA=10\) cm and angle \(\angle APB=60^\circ\), find the length of chord AB.
21. If radii of the two concentric circles are 15cm and 17cm, then find the length of the chord of the larger circle which is tangent to one other.
22. Points P, Q, R, are on a circle with centre O such that the figure OPQR is a rhombus. If the area of the rhombus is $32\sqrt{3}\text{cm}^2$, find the radius of the circle.

23. In figure, XP and XQ are tangents from X to the circle with centre O. R is a point on the circle and ARB is another tangent. Prove that $XA + AR = XB + BR$.

24. A circle is inscribed in a triangle ABC having sides 8cm, 10cm and 12cm as shown in the figure. Find AD, BE and CF.

25. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre.

26. AB is a chord of length 9.6cm of a circle with centre O and radius 6cm. If the tangents at A and B intersect at point P then find the length PA.

27. The in-circle of a $\Delta ABC$ touches the sides BC, CA & AB at D, E and F respectively. If AB = AC, prove that BD = CD.

28. PQ and PR are two tangents drawn to a circle with centre O from an external point P. Prove that $\angle QPR = 2 \angle OQR$.

29. If a, b, c are the sides of a right triangle where c is the hypotenuse, then prove that radius $r$ of the circle touches the sides of the triangle is given by $r = \frac{(a+b-c)}{2}$.

30. Prove that the lengths of the tangents drawn from an external point to a circle are equal.

31. Prove that the radius of a circle is perpendicular to the tangent at the point of contact.
ACTIVITIES/PROJECTS

ACTIVITY 1

Angles in the same segment

Objective
To verify that the angles in the same segment of a circle are equal, using the method of paper cutting, pasting and folding.

Pre-requisite knowledge
Geometrical meaning of segment of a circle.

Materials required
coloured papers, pair of scissors, gum, scale, compass, pencil, carbon papers or tracing papers

Procedure

1. Draw a circle of any radius with centre O and cut it.
2. Paste the cutout on a rectangular sheet of paper. [Fig 7(a)].
3. Fold the circle in any way such that a chord is made. Draw the line segment AB. [Fig 7(b)].
4. Take two points P and Q on the circle in the same segment. [Fig 7(c)].
5. Form a crease joining AP. Draw AP. [Fig 7(d)].
6. Form a crease joining BP. Draw BP. [Fig 7(e)].
7. \( \angle APB \) is formed in the major segment. [Fig 7(f)]
8. Form a crease joining AQ. Draw AQ. [Fig 7(g)]
9. Form a crease joining BQ. Draw BQ. [Fig 7(h)]
10. \( \angle APB \) and \( \angle AQB \) are formed in the major segment. [Fig 7(i)]
11. Make replicas of \( \angle APB \) and \( \angle AQB \) using carbon paper or tracing paper. [Fig 7(j)]
12. Place the cutout of \( \angle APB \) on the cutout of \( \angle AQB \). What do you observe?

Observations

Students will observe that
1. \( \angle APB \) and \( \angle AQB \) are angles in the same segment.
2. \( \angle AQB \) covers \( \angle APB \) exactly. Therefore \( \angle APB = \angle AQB \)

Learning outcome

Students become more familiar with this theorem (proved in the classroom) through an activity.

Remarks

1. The teacher may ask the student to perform the activity by taking different points P and Q including those very close to points A and B.

2. The teacher may ask the student to perform the activity using point P in one segment and Q in the other segment and note that the angles in the segments are not equal, except in the case when the chord AB is a diameter of the circle. They will find that the two angles are supplementary Angles.
ACTIVITY -2

Angle in a semicircle, major segment and minor segment

Objective
To verify, using the method of paper cutting, pasting and folding that
(a) the angle in a semicircle is a right angle,
(b) the angle in a major segment is acute,
(c) the angle in a minor segment is obtuse.

Pre-requisite knowledge
Concept of right angle, acute angle, obtuse angle, linear pair axiom.

Procedure for (a)
1. Draw a circle of any radius with centre O on a coloured sheet of a paper and cut it.[Fig 8a(a)]
2. Form a crease passing through the centre O of the circle. Diameter AB is obtained. Draw AB. [Fig 8a(b)]
3. Take a point P on the semicircle.
4. Form a crease joining AP. Draw AP. [Fig 8a(c)]
5. Form a crease joining BP. Draw BP. [Fig 8a(d)]
6. Make two replicas of \(\angle APB\); call them \(\angle A1P1B1\) and \(\angle A2P2B2\). [Fig 8a(e)]
7. Place the two replicas adjacent to each other such that A1P1 and P2B2 coincide with each other as shown in Fig 8a(f).

Observations
Students will observe that the two line segments P1A1 and P2B2 lie on a straight line.
Therefore, \(\angle A2P2B2 + \angle B1P1A1 = 180^\circ\)
But \(\angle A2P2B2\) and \(\angle B1P1A1\) are replicas of \(\angle APB\).
i.e. \(2\angle APB = 180^\circ\)
i.e. \(\angle APB = 90^\circ\)

Procedure for (b)
1. Draw a circle of any radius with centre O on a coloured sheet of paper and cut it.
2. Paste the cutout on a rectangular sheet of paper. [Fig 8b(a)]
3. Fold the circle in such a way that a chord AB is obtained. Draw AB. [Fig 8b(b)]
4. Take a point P in the major segment.
5. Form a crease joining AP. Draw AP. [Fig 8b(c)]
6. Form a crease joining BP. Draw BP. [Fig 8b(d)]
7. Make a replica of \(\angle APB\). [Fig 8b(e)].
8. Place the replica of \(\angle APB\) on a right angled \(\triangle DEF\) such that BP falls on DE. [Fig 8b(f)]. What do you observe?

Materials required
coloured papers, a pair of scissors, gum, scale, compass, pencil, carbon paper or tracing paper, cut out of right angled triangle.

Observations
Students will observe that
1. \(\angle BPA\) does not cover \(\triangle DEF\) completely. [Fig 8b (f)]
2. \(\angle BPA\) is smaller than the \(\angle DEF\).
3. \( \angle DEF \) is 90\(^\circ\).
4. Therefore, \( \angle BPA \) is acute.
5. \( \angle BPA \) is an angle in the major segment

**Procedure for (c)**
1. Draw a circle of any radius with centre O on a coloured sheet of paper and cut it.
2. Paste the cutout on a rectangular sheet of paper. [Fig 8c(a)]
3. Fold the circle in such a way that a chord AB is obtained. Draw AB. [Fig 8c(b)]
4. Take a point P in the minor segment.
5. Form a crease joining AP. Draw AP. [Fig 8c(c)]
6. Form a crease joining BP. Draw BP. [Fig 8c (d)]
7. Make a replica of \( \angle APB \). [Fig 8c (e)]
8. Place the right angled \( \square DEF \) on the replica of \( \angle APB \) such that DE falls on BP. [Fig 8c (f)] What do you observe?

**Learning outcome**
Students develop familiarity with the fact that the angle in a semicircle is right angle, the angle in a major segment is acute angle and the angle in a minor segment is obtuse angle.

**Remark**
The teacher may point out that for a given chord AB, the obtuse angle in the minor segment and the acute angle in the major segment are supplementary angles. The students may be asked to verify this by taking appropriate cutouts of the angles.
ACTIVITY 3

Cyclic Quadrilateral Theorem

Objective
To verify, using the method of paper cutting, pasting and folding that
1. The sum of either pair of opposite angles of a cyclic quadrilateral is 180°.
2. In a cyclic quadrilateral the exterior angle is equal to the interior opposite angle.

Pre-requisite knowledge
1. Meaning of cyclic quadrilateral, interior opposite angle
2. Linear pair axiom

Materials required
coloured papers, pair of scissors, ruler, sketch pen, carbon paper or tracing paper.

Procedure
1. Draw a circle of any radius on a coloured paper and cut it.
2. Paste the cutout on a rectangular sheet of paper. [Fig 9(a)].
3. By paper folding get chords AB, BC, CD and DA.
4. Draw AB, BC, CD and DA. Cyclic quadrilateral ABCD is obtained [Fig 9(b)].
5. Make a replica of cyclic quadrilateral ABCD using carbon paper / tracing paper. [Fig 9(c)]
6. Cut the quadrilateral cutout into four parts such that each part contains one angle
   i.e. \( \angle A, \angle B, \angle C \text{ and } \angle D \). [Fig 9(d)]
7. Place \( \angle A \) and \( \angle C \) adjacent to each other. What do you observe?
8. Produce AB to form a ray AE. Exterior angle \( \angle CBE \) is formed. [Fig 9(f)]
9. Place the replica of D on \( \angle CBE \). [Fig 9(g)] What do you observe?

Learning outcome
Students develop geometrical intuition of the result that
1. Opposite angles of a cyclic quadrilateral are supplementary.
2. Exterior angle to a cyclic quadrilateral is equal to the interior opposite angle.

Remarks
1. The teacher may ask the students to perform the activity for the other pair of angles
   (i.e. \( \angle B \) and \( \angle D \)) and for the other exterior angles also.
2. The teacher should point out that this theorem is true only for a cyclic quadrilateral.
   Students may be asked to perform a similar activity for a non-cyclic quadrilateral.
ACTIVITY -4

Tangents are equally inclined to the line segment joining the centre with the external point.
1. Cut out the two triangles, ΔOPA and ΔOPB, so formed [figure 3].
2. Colour the two triangles with different colours.
3. Put one triangle on the other.

Observation:
You will observe that two triangles are congruent to each other (i.e., one triangle exactly superimposes the other) with the following (angle) correspondence.
\[ \angle OPA = \angle OPB, \] (Proves the required result)
\[ \angle PAO = \angle PBO, \]
\[ \angle AOP = \angle BOP \]

Hence, tangents drawn from an external point, are equally inclined to the line segment joining the centre with that point.

**Result :**

Tangents drawn from an external point are equally inclined to the line segment joining the centre with the external point.

**Note :** The tangent at any point of a circle is perpendicular to the radius through the point of contact, i.e.,

\[ \angle OAP = \angle OBP = 90^\circ. \]

**Activity - 5**

**Tangents drawn from an external point**

**Objective**

To verify using the method of paper cutting, pasting and folding that the lengths of tangents drawn from an external point are equal.

**Pre-requisite knowledge**

Meaning of tangent to a circle

**Materials required**

coloured papers, pair of scissors, ruler, sketch pens, compass, pencil.

**Procedure**

1. Draw a circle of any radius on a coloured paper and cut it. Let O be its centre.
2. Paste the cutout on a rectangular sheet of paper.[Fig 10(a)]
3. Take any point P outside the circle.
4. From P fold the paper in such a way that it just touches the circle to get a tangent PA (A is the point of contact). [Fig 10(b)]. Join PA.
5. Repeat step 4 to get another tangent PB to the circle (B is the point of contact). [Fig 10(c)]. Join PB.
6. Join the centre of the circle O to P, A and B. [Fig 10(d & e)]
7. Fold the paper along OP. [Fig 10(f)] What do you observe?

**Observations**

Students will observe that

1. \( \triangle OPA \) and \( \triangle OPB \) completely cover each other.
2. Length of tangent PA = Length of tangent PB.
Learning outcome
Students learn how to get tangents from an external point to a circle using paper folding and verify the theorem.

Remark
The teacher may ask the students to perform the activity by taking point P (external point) at different locations.

Research:
Explore the relation between a triangle and a circle and try to find the answer to following questions:

A) How many cases are possible when there is 0 points of intersection?
B) How many cases are possible when there is 1 points of intersection?
C) How many cases are possible when there are 2 points of intersection?
D) How many cases are possible when there are 3 points of intersection?
E) How many cases are possible when there are 4 points of intersection?
F) How many cases are possible when there are 5 points of intersection?
G) How many cases are possible when there are 6 points of intersection?
H) CROSS-CURRICULAR LINK:

A potter found a piece of beautiful circular plate. He wants to produce replica of original size. How can he determine the original size of the plate?

POWER POINT PRESENTATIONS

WEB LINKS

Worksheets

- http://www.teach-nology.com
- A demo on tangents to a circle and PPT
- https://www.youtube.com/watch?v=bY3Jsi-tjrk
- http://www.learnnext.com/CBSE-Class-X-Maths/Lesson-Tangents-to-a-Circle.htm#
- http://www.learnnext.com/CBSE-Class-X-Maths/Lesson-Constructions.htm#
- http://schools.aglasem.com/1202
- https://www.youtube.com/watch?v=COiGT5t4Uyc
- https://www.youtube.com/watch?v=eqs0VaBvg3Y&list=PL56C5CE7748C6224B&index=2
- https://www.youtube.com/watch?v=i7BX-UPxEn8&index=3&list=PL56C5CE7748C6224B
- https://www.youtube.com/watch?v=2BCyexezkl&index=4&list=PL56C5CE7748C6224B
- https://www.youtube.com/watch?v=lv8mBuuuu94&list=PL56C5CE7748C6224B&index=5
- https://www.youtube.com/watch?v=PFzph3ASgi4&index=6&list=PL56C5CE7748C6224B&index=7
- PPT
- http://www.slideshare.net/AkshayFegade/cbse-10th-circles
- http://www.authorstream.com/Presentation/vampireadi-1587680-circles/
- https://www.youtube.com/watch?v=i7BX-UPxEn8
- https://www.youtube.com/watch?v=BrNA6G4THUo
- https://www.youtube.com/watch?v=WejESWLYrps
- https://www.youtube.com/watch?v=AFCe5DPkxJ1
- Properties of tangents:
- http://www.youtube.com/watch?v=j0yuSLI8QGc&feature=related
- http://www.youtube.com/watch?v=WejESWLyrps&feature=related
- http://www.youtube.com/watch?v=1pPGNu-ZM-0&feature=related
  - Solving questions on circles and tangents
- http://www.youtube.com/watch?v=JgF29pILDD8&feature=related
- http://www.youtube.com/watch?v=OfIvTh2gqA8&feature=related
- http://www.youtube.com/watch?v=uHal7hqIFjw&feature=related
- http://www.youtube.com/watch?v=p00RIUvg1K4&feature=related
- http://www.youtube.com/watch?v=tIEAQjrv8Os&feature=related
  - Definition of tangent
- http://www.youtube.com/watch?v=Ut_rKPeh-JE&feature=player_embedded
  - Radius and tangent of a circle
- http://bcove.me/axis2i2q
  - Tangent Segments to a circle (length of tangent)
- http://bcove.me/t8tsw680
  - Inscribed angle
- http://bcove.me/ykurgkk5
  - Angles in semicircles and chords to tangents
- http://bcove.me/jxdtwo48
  - Common Internal and External Tangents
- http://www.youtube.com/watch?v=FmXxPMFifSs&feature=player_embedded
  - Finding length of tangent when radius is given
  - Interactive Sheet on tangent to a circle
- http://www.mathwarehouse.com/geometry/circle/tangent-to-circle.php#
CHAPTER-11

CONSTRUCTIONS

INTRODUCTION

Basic Concepts

- Division of line segment in a given ratio internally and externally
- Tangent to a circle from a point outside it.
- Construction of triangle similar to given triangle.

We have already done certain constructions e.g. bisecting an angle, drawing perpendicular bisector of a line segment, some constructions of triangles, etc, with ruler and compass in previous class. Now we shall do some more constructions.

EXPECTED LEARNING OUTCOMES

1. Correct use of Mathematical instruments
2. Drawing a line segment and an angle as per the given data
3. To divide the given line segment in the given ratio accurately.
4. Neatness and accuracy in drawing
5. The concept of similar triangles.
6. To Construct a triangle as per the criteria conditions given
7. To construct similar triangle for a given triangle as per the given ratio.
8. To know that when the ratio is a proper fraction then the similar triangle lies inside the given triangle and when improper then the similar triangles lie outside the given triangle.
9. To construct tangents to a circle from an external point given.
10. To read and to understand the verbal sum given to construct and draw rough figures
GRADED EXERCISES

LEVEL I

1. Draw a line segment AB = 7 cm. Take a point P on AB such that AP : PB = 3 : 4.
2. Draw a line segment PQ = 10 cm. Take a point A on PQ such that PA/PQ ::2/5 Measure the length of PA and AQ.
3. Construct a ΔABC in which BC = 6.5 cm, AB = 4.5 cm and ∠ACB = 60°.Construct another triangle similar to ΔABC such that each side of new triangle is 4/5 of the corresponding sides of ΔABC.
4. Draw a triangle XYZ such that XY = 5 cm, YZ = 7 cm and ∠XYZ = 75°.Now construct a ΔX'YZ' ~ ΔXYZ with its sides 3/2 times of the corresponding sides of ΔXYZ.
5. Draw ΔPQR in which ∠Q = 90°, PQ = 6 cm, QR = 8 cm. Construct ΔP'QR' ~ ΔPQR with its sides equal to 2/3rd of corresponding sides of ΔPQR.
6. Draw a circle of radius 4 cm with centre O. Take a point P outside the circle such that OP = 6 cm. Draw tangents PA and PB to the circle. Measure the lengths of PA and PB.

7. Draw a circle of radius OP = 3 cm. Draw \( \perp \) POQ = 45° such that OQ = 5 cm. Now draw two tangents from Q to given circle.

8. Draw a line segment AB = 8 cm and divide it in the ratio 4:3.

9. Divide a line segment of 7 cm internally in the ratio 2:3.

10. Draw a circle of radius 4 cm. Take a point P on it. Draw tangent to the given circle at P.

11. Construct an isosceles triangle whose base 7.5 cm and altitude is 4.2 cm.

**LEVEL II**

1. Construct an isosceles triangle whose base is 8 cm and altitude 5 cm and then construct another triangle whose sides’ are 3/4 times the corresponding sides of the given triangle.

2. Draw an isosceles \( \triangle \) ABC with AB = AC and base BC = 7 cm and vertical angle is 120°. Construct \( \triangle \) AB’C’ ~ \( \triangle \) ABC with its sides 4/3 times of the corresponding sides of \( \triangle \) ABC

3. Draw an equilateral triangle PQR with side 5 cm. Now construct \( \triangle \) PQ’R’ such that \( \frac{PQ'}{PQ} = \frac{1}{2} \). Measure PQ’.

4. Draw a line segment AB = 8 cm. Taking AB as diameter a circle is drawn with centre O. Now draw OP \( \perp \) AB. Through P draw a tangent to the circle.

5. Draw a circle with centre O and radius 3.5 cm. Draw two tangents PA and PB from an external point P such that \( \perp \) APB = 45°. What is the value of \( \perp \) AOB + \( \perp \) APB

6. Draw a line segment AB = 9 cm. Taking A and B as centres draw two circles of radius 5 cm and 3 cm respectively. Now draw tangents to each circle from the centre of the other

7. Draw a circle of diameter 7 cm. Draw a pair of tangents to the circle, which are inclined to each other at an angle of 60°.

8. Construct a triangle of sides 4 cm, 5 cm and 6 cm and then triangle similar to it whose side is 2/3 of corresponding sides of the first triangle.

9. Construct a triangle similar to a given \( \triangle \) ABC such that each of its sides is 2/3rd of the corresponding sides of \( \triangle \) ABC. It is given that AB = 4 cm BC = 5 cm and AC = 6 cm also write the steps of construction.

10. Draw a right triangle ABC in which \( \perp \) B = 90° AB = 5 cm, BC = 4 cm then construct another triangle ABC whose sides are 5/3 times the corresponding sides of \( \triangle \) ABC.

11. Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°.

12. Draw a circle of radius 5 cm from a point 8 cm away from its centre construct the pair of tangents to the circle and measure their length.

13. Construct a triangle PQR in which QR = 6 cm \( \perp \) Q = 60° and \( \perp \) R = 45°. Construct another triangle similar to \( \triangle \) PQR such that its sides are 5/6 of the corresponding sides of \( \triangle \) PQR.

**LEVEL III**

1. Construct a right angled triangle in which base is 2 times of the perpendicular. Now construct a triangle similar to it with base 1.5 times of the original triangle

2. Draw a circle of radius 4 cm. Now draw a set of tangents from an external point P such that the angle between the two tangents is half of the central angle made by joining the points of contact to the centre

3. Draw a circle of radius 3.5 cm with centre O. Take point P such that OP = 6 cm. OP cuts the circle at T. Draw two tangents PQ and PR. Join Q to R. Through T draw AB parallel to QR such that A and B are point on PQ and PR.
4. Draw a circle with centre O and radius 3.5 cm. Take a horizontal diameter. Extend it to both sides to point P and Q such that OP = OQ = 7 cm. Draw tangents PA and QB one above the diameter and the other below the diameter. Is PA || BQ.

5. Draw a circle of radius 8 cm. From a point 17 cm away from its centre construct a pair of tangents to the circle and measure their tangents.

6. Draw a line segment AB = 7.5 cm and locate a point P on AB such that AP = 3/7 AB. Give justification of the construction.

**VALUE BASED QUESTIONS**

1. Construct a triangle similar to a given ΔABC, where AB = 4 cm, BC = 6 cm and ∠ABC = 60°, such that each of its sides is \( \frac{5}{7} \) of the corresponding sides of the ΔABC. What qualities of Gandhiji would you like to construct within you.

2. Draw a line segment of length 8 cm divided it in the ratio 3:4. Dividing Joint families into nuclear families is good or bad.

3. Draw a circle of radius 5 cm. Draw tangents from end points of its diameter. What do you observe?
   
   If each tangent represents the quality of a human being, find out the qualities that should be adopted for a better human being.

4. In a school, the Xth class students were awarded full marks for the following question in the paper of Mathematics even when none of the students attempted it.

5. Construct a circle of radius 6 cm and draw the tangents CT₁ and CT₂ from a point outside the circle such that CT₁ = 9 cm and CT₂ = 10 cm.

   (i) Is the given construction is correct?

   (ii) If the point C is at the distance of 10 units from the centre, then draw the tangents from C and measure their lengths.

   (iii) By doing so, which value is displayed by the school administration?

**ERROR ANALYSIS**

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<tr>
<th>Sno</th>
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<th>REMEDIATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Given scale was ( \frac{3}{4} ) but many students took it as ( \frac{4}{3} ) and construct ΔAB’C’ bigger than Δ ABC</td>
<td>Difference between external division and internal division (i.e.) scale factor ( \frac{4}{3} ) or ( \frac{3}{4} ) should be made clear by taking different examples</td>
</tr>
<tr>
<td>2</td>
<td>Many students could not draw parallel lines correctly using compass, while constructing the similar triangles</td>
<td>Practice to be given for drawing parallel lines with compass only</td>
</tr>
<tr>
<td>3</td>
<td>Students did not understand the similarity of Δ ABC to Δ BAC or Δ ABC to Δ A’BC</td>
<td>Practice has to be given for constructing ΔA’B’C’ ~ ΔABC (common point A) ΔA’BC’ ~ Δ ABC (common point B) and Δ A’B’C ~ ΔABC (common point C)</td>
</tr>
<tr>
<td>4</td>
<td>Application of similarity results are not proper</td>
<td>Correct and appropriate use of similarity result</td>
</tr>
</tbody>
</table>
While constructing similar figures should be discussed and practiced

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Some students draw untidy construction</td>
</tr>
<tr>
<td>6</td>
<td>Some students drew parallel lines using ruler only</td>
</tr>
</tbody>
</table>

**QUESTION BANK**

1. Draw an equilateral triangle ABC of each side 4 cm. Construct a triangle similar to it and of scale factor $3/5$. Is the new triangle also an equilateral?
2. Draw a line segment of length 7 cm. Find a point P on it which divides it in the ratio 3:5.
3. Draw a right triangle ABC in which BC = 12 cm, AB = 5 cm and $\angle B = 90^\circ$. Construct a triangle similar to it and of scale factor $2/3$. Is the new triangle also a right triangle?
4. Draw a triangle ABC in which BC = 6 cm, CA = 5 cm and AB = 4 cm. Construct a triangle similar to it and of scale factor $5/3$.
5. Construct a tangent to a circle of radius 4 cm from a point which is at a distance of 6 cm from its centre.
6. Given a rhombus ABCD in which AB = 4 cm and $\angle ABC = 60^\circ$, divide it into two triangles say, ABC and ADC. Construct the triangle AB'C' similar to $\triangle ABC$ with scale factor $2/3$. Draw a line segment CD' parallel to CD where D' lies on AD. Is AB'C'D' a rhombus? Give reasons.
7. Two line segments AB and AC include an angle of $60^\circ$ where AB = 5 cm and AC = 7 cm. Locate points P and Q on AB and AC, respectively such that AP = $3/4$ AB and AQ = $1/4$ AC. Join P and Q and measure the length PQ.
8. Draw a parallelogram ABCD in which BC = 5 cm, AB = 3 cm and $\angle ABC = 60^\circ$, divide it into triangles BCD and ABD by the diagonal BD. Construct the triangle BD'C' similar to $\triangle BDC$ with scale factor $4/3$. Draw the line segment D'A' parallel to DA where A' lies on extended side BA. Is A'BC'D' a parallelogram?
9. Draw two concentric circles of radii 3 cm and 5 cm. Taking a point on outer circle construct the pair of tangents to the other. Measure the length of a tangent and verify it by actual calculation.
10. Draw an isosceles triangle ABC in which AB = AC = 6 cm and BC = 5 cm. Construct a triangle PQR similar to ABC in which PQ = 8 cm. Also justify the construction.
11. Draw a triangle ABC in which AB = 5 cm, BC = 6 cm and $\angle ABC = 60^\circ$. Construct a triangle similar to ABC with scale factor $5/7$. Justify the construction.
12. Draw a circle of radius 4 cm. Construct a pair of tangents to it, the angle between which is $60^\circ$. Also justify the construction. Measure the distance between the centre of the circle and the point of intersection of tangents.
13. Draw a triangle ABC in which AB = 4 cm, BC = 6 cm and AC = 9 cm. Construct a triangle similar to $\triangle ABC$ with scale factor $3/2$. Justify the construction. Are the two triangles congruent? Note that all the three angles and two sides of the two triangles are equal.
ACTIVITIES/PROJECTS

Converting a Triangle into a Square

On a thick sheet of paper, construct an equilateral $\triangle ABC$. Divide the triangle $ABC$ into four pieces as shown in the figure. Here, $AD = BD$, $AE = CE$, $BF = \frac{1}{4} BC$, $CG = \frac{1}{4} BC$. $DH \perp EF$ and $GI \perp EF$.

![Diagram of triangle divided into four pieces]

Cut the pieces out and rearrange the pieces to form a square.

Converting a Rectangle into a Square

On a thick sheet of paper, draw a rectangle of dimensions $5 \text{ cm} \times 2 \text{ cm}$.

![Rectangle diagram]

Using three straight cuts, divide the rectangle into 5 pieces such that these pieces when rearranged give a square.

Making Rectangle From Squares

On thick sheets of paper, draw squares of sides $1 \text{ cm}$, $4 \text{ cm}$, $7 \text{ cm}$, $8 \text{ cm}$, $9 \text{ cm}$, $10 \text{ cm}$, $14 \text{ cm}$, $15 \text{ cm}$ and $18 \text{ cm}$. Cut out each square. Now rearrange these square pieces to form a rectangle. Paste the arrangement on a sheet of paper.

Matchstick Puzzle

A $3 \times 3$ array of matchsticks is given. From this array remove exactly four matchsticks to get five identical squares.

![Matchstick puzzle diagram]
Extension Activities:

1. Given a quadrilateral inscribed in a circle, how will you find the center and radius of the circle?
2. Given a circle inscribed in a quadrilateral, how will you find the center and radius of the circle?

POWER POINT PRESENTATIONS

SOLUTIONS TO CHAPTER - 11: CONSTRUCTIONS

STEPS TO VIEW THE SOLUTIONS USING ROBOCOMPASS

- Robocompass is a free online software
- The solutions to the text books questions can be viewed online only
- Hence the computer has to be connected to internet to view the solutions. An internet connection with good speed is required for the same
- Individual link is given against each question of all the chapters of geometric construction in classes VI to X NCERT Text book

- Click the link against the question to view the solution

CLICK TO OPEN USER GUIDE PPT

CLICK TO OPEN USER GUIDE IN PDF
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ADVANTAGES OF ROBOCOMPASS

✓ The animations are exactly as we do using a physical straightedge, compass and protractor
✓ The construction can be viewed any number of times
✓ A particular step of construction can be viewed to get a better understanding
✓ Colourful presentation of arcs and lines will increase interest among the learner Interesting projects can be given to the students.
✓ Interesting projects can be given to the students

RESTRICTIONS IN ROBOCOMPASS

✓ Robocompass is free online software. Hence internet connection is required to view the files designed using Robocompass
✓ The present edition of Robocompass does not support some Mathematics Symbols like: \( \angle, ^\circ \), etc.
✓ Usually perpendicular bisectors will be denoted by dotted lines. Robocompass does not support construction of dotted lines.
✓ Labelling a vertex as A’, B’, A_1, B_1 is not possible using Robocompass
✓ All these restrictions may be removed in the updated versions of Robocompass
# More Information about Robocompass

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<th>S.No.</th>
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<th>Link to Open the Resource</th>
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<td>2</td>
<td>Robocompass Commands</td>
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<td>3</td>
<td>Getting Started with Robocompass YouTube Video (Internet Required)</td>
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**Web Links**

- [https://www.youtube.com/watch?v=vvKMIjVSvM&list=PL50809012BC13C5D8](https://www.youtube.com/watch?v=vvKMIjVSvM&list=PL50809012BC13C5D8)
- [https://www.youtube.com/watch?v=xGHAyVdxwiWQ&list=PL50809012BC13C5D8&index=2](https://www.youtube.com/watch?v=xGHAyVdxwiWQ&list=PL50809012BC13C5D8&index=2)
- [http://www.mathopenref.com/consttangents.html](http://www.mathopenref.com/consttangents.html)
AREAS RELATED TO CIRCLES

INTRODUCTION

We come across many objects in our daily life that are circular in shape like Cycle wheels, wheel barrow, dartboard, round cake, papad, drain cover, various designs, bangles, brooches, circular paths etc.

This chapter deals in calculating the area and perimeter of segments and sectors. It also deals with how to find the areas of some combinations of plane figures involving circles or their parts.

EXPECTED LEARNING OUTCOMES

1. To calculate the area of a circle.
2. To calculate the circumference of a circle.
3. To calculate the length of an arc of a sector of a circle.
4. To calculate the area of a sector of a circle.
5. To calculate the area of a segment of a circle.
6. To calculate the area of combinations of plane figures.
ONCEPT MAP

Areas Related to circles

Terms related to circles

- circumference of a circle
- perimeter of a semicircle
- perimeter of a quadrant

Area of a circle

sector of a circle

- to calculate

segment of a circle

- to calculate
- to calculate

perimeter of a segment

Area of a segment

Areas of combinations of plane figures

priya-areas related to circles.vue.new.vue
GRADED EXERCISES

LEVEL – I

1. Find the perimeter of the protractor if its radius is 7cm. (1m)
2. Find the circumference of a circle whose area is 16 times the area of the circle with diameter 7cm. (2m)
3. Find the perimeter of a quadrant of a circle of radius 3.5 cm. (2m)
4. If the area of the minor segment of a circle with radius 7cm is $54cm^2$, find the area of the corresponding major segment. (1m)
5. In a circle of radius 21cm, find the length of the arc which subtends an angle of $60^0$. (1m)
6. The radii of two circles are 8cm and 6cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles. (2m)
7. Find the area of the shaded region in the given figure if AB=8CM, AC=10CM and BC=6CM.

8. Find the area of the shaded region if the radii of two concentric circles are 7cm and 3.5cm. (2m)
9. Find the area of the shaded region. (2m each)
10. Find the radius of the circle whose area and circumference are equal. (2m)

LEVEL -2

1. The perimeter of a sector of a circle of radius 5.7 m is 27.2 m. Find the area of the sector. (2m)
2. The area enclosed between the two concentric circles is $770 cm^2$. If the radius of the outer circle is 21cm, calculate the radius of the inner circle. (3m)
3. PQRS is a diameter of a circle of radius 6 cm. The lengths PQ, QR and RS are equal, semi-circles are drawn on PQ and QS as diameters as shown in figure. Find the perimeter and area of the shaded region. (3m)
4. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find:
   (i) Area of the minor sector 
   (ii) Area of the minor segment 
   (iii) Area of major sector 
   (iv) Area of major segment (use \( \pi = 3.14 \))

5. A wheel of diameter 42 cm, makes 240 revolutions per minute. Find:
   (i) the total distance covered by the wheel in one minute.
   (ii) the speed of the wheel in km/hr.

6. In the following figure, ABC is an equilateral triangle. Circles are drawn with vertices of the triangle ABC as centers so that every circle touches the remaining two. If the perimeter of the triangle ABC is 84 cm. Find: (i) the area of sector, inside the triangle, of each circle

7. Find the area of shaded region, if the side of square is 28 cm and radius of the sector is \( \frac{1}{2} \) the length of side of square.

8. In the adjoining figure, ABCD is a square of side 6 cm. Find the area of the shaded region.
LEVEL -3

1. The adjoining figure shows the cross-section of a railway tunnel. The radius of the tunnel is 3.5m (i.e., OA=3.5m) and AOB=90°. Calculate:
   i. the height of the tunnel.
   ii. the perimeter of its cross section, including base.
   iii. the area of the cross-section
   (3m)

![Diagram of cross-section of a railway tunnel]

2. In the adjoining figure, ABCD is a rectangle with sides 4cm and 8cm. Taking 8cm as the diameter, two semicircles are drawn. Find the area overlapped by the two semicircles.
   (4m)

![Diagram of rectangle and semicircles]

3. In the figure, ABCD is a square of side 8 cm. CBED and ADFB are quadrants of circle. Find the area of the shaded region. (Use π = 3.14)
   (4m)

![Diagram of square and quadrants]

4. In the figure, ABC is a quadrant of a circle of radius 14 cm and a semi circle is drawn with BC as diameter. Find the area of shaded region.
   (4m)
5. ABC is a right angled triangle in which angle $A = 90^0$. Find the area of the shaded region if $AB = 6$ cm, $BC = 10$ cm & I is the centre of the circle of triangle ABC. (4m)

6. In the given figure, O is the centre of the bigger circle, and AC is its diameter. Another circle with AB as diameter is drawn. If $AC = 54$ cm and $BC = 10$ cm, find the area of the shaded region. (4m)

7. In given Fig., ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the perimeter of the shaded region. (4m)

8. The boundary of the shaded portion in the adjoining figure consists of four semi circles and two quarter-circles. Find the length of the boundary and the area of the shaded portion, if $OA=OB=OC=OD=14$ cm. (4m)
VALUE BASED QUESTIONS

1. Ramu has a lawn in one corner of his house in the shape shown in the figure (shaded area). The corner of the house is the center of the sector. He has installed a sprinkler system to water his lawn, with the sprinkler head located at the center, instead of using a hose pipe. The central angle of the sector is $216^\circ$ and the inner and outer radii are 2m and 5m respectively.
   i. Find the area of the lawn to be watered.
   ii. Comment about the method selected by Ramu to water his lawn.

2. In the figure shown, Mr. Somu has his office located at A and his house located at B. Somu drives his car two days in a week and rides his cycle the remaining 3 days to get to his office. AOB is a sector of a circle with center O and central angle $60^\circ$, with radius 3.5 km. Path AOB is the route for driving the car and Path ACB is a cycle-only track.
   By which means of transport does Mr. Somu travel a longer distance during the 5 day week? Justify.
   What do you think about Mr. Somu’s preference for cycling? What are the benefits cycles riding over car driving?
3. To expand its business, a company wants to buy agricultural land from farmers in the shape of a right triangle ABC as shown in the figure. The Government instructs the company to compensate the farmers by giving the land in the shape shown as Area 1 and Area 2. Are the farmers being compensated fairly? Justify. What do you think about the need to protect farmers’ interest in our country?

4. Farmer has two types of field in the form of triangle and rectangle. Geeta is allowed to cut the grass of triangular field (shaded position) and Vijay is allowed to cut the grass of rectangular field (shaded portion) in the following manner. Calculate the areas of both shaded portions? Which value is depicted?

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<td>Students have confusion between sector and segment</td>
<td>Concept should be made clear by activities</td>
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<tr>
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<td>Finding perimeter of sector, students do not take the radius into consideration</td>
<td>Students should draw the figure and note the data given to get the answer correctly</td>
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<tr>
<td>3</td>
<td>Students get confused while using the formula for area of sector and length of arc of sector</td>
<td>Emphasize and practice the correct usage of formula</td>
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<tr>
<td>4</td>
<td>While finding the area of shaded region, students forget to subtract the common area overlapped</td>
<td>Through practice to be given for such sums</td>
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</table>
1. How many times will the wheel of a car rotate in a journey of 2002 m, if the radius of the wheel is 49 cm.

2. In Radhika’s house there is a flower pot. The sum of radii of circular top and bottom of the flower pot is 140 cm and the difference of their circumferences is 88 cm. Find the diameters of the circular top and bottom.

3. Two circles touch internally. The sum of their areas is 116 cm² square cm and distance between their centres is 6 cm. Find the radii of the circles.

4. In the given figure, OPQR is a rhombus whose three vertices P, Q, R lie on a circle of radius 8 cm. Find the area of the shaded region.

5. A larger wheel of diameter 50 cm is attached to a smaller wheel of diameter 30 cm. Find the number of revolutions made by the smaller wheel, when the larger one makes 15 revolutions.

6. From a circular piece of cardboard with radius 1.26 m, a sector with central angle 40° has been removed. Find (i) Area of the portion removed (iii) Perimeter of the sector removed

7. In the given figure, if O is the centre of the circle AB = CD and OB = 3 cm, then find the area of the shaded portion.
8. In the given figure, SR is the diameter of length 12 cm and SP = PQ = QR, then find the area and perimeter of the shaded region.

9. A child draws the figure of an aeroplane as shown. Here, the wings ABCD and FGHI are parallelograms, the tail DEF is an isosceles triangle, the cockpit CKI is a semicircle and CDFI is a square. In the given figure, BP \perp CD, TT \perp CD, HQ \perp FI and EL \perp DF. If CD = 8 cm, BP = HQ = 4 cm and DE = EF = 5 cm, find the area of the whole figure. Take \pi = 3.14.

10. In the figure, ABCDEF is any regular hexagon. With different vertices A, B, C, D, E and F as the centres of circles with same radius \( r \) are drawn. Find area the of the shaded portion.

11. In the figure, arcs are drawn by taking vertices A, B and C of an equilateral triangle of side 10 cm, to intersect the sides BC, CA and AB at their respective mid-points D, E and F. Find the area of the shaded region (Use \( \pi = 3.14 \))
ACTIVITIES/PROJECTS

ACTIVITY-1

Objective: To derive the formula for area of sector of a circle.

Materials Required: Glaze paper, geometry box, a pair of scissors, fevistick etc.

Procedure:

1. Draw some circles of any radius (say 3 cm) on a glaze paper. Cut these out and paste them on a drawing sheet.
2. Mark two points P and Q on the circumference of one of the circles. Join OP and OQ. The region OPQ is called the sector of a circle. Mark $\angle POQ = \theta$ POQ is the angle of the sector.
3. Now on other circles, make different sectors of $45^\circ$, $60^\circ$, $90^\circ$ and $120^\circ$
4. Circle $C_1$ with sector of $45^\circ$, circle $C_2$ with sectors of $60^\circ$, circle $C_3$ with sector of $90^\circ$ and circle $C_4$ with sector of $120^\circ$.

1. Calculate the areas of sectors of $C_1$, $C_2$, $C_3$ and $C_4$, record your observations in the following table.

<table>
<thead>
<tr>
<th>Circle</th>
<th>Angle of the sector</th>
<th>No. of equal sectors in a circle</th>
<th>Area of each sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_3$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_4$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**ACTIVITY-2**

Objective: To prepare a medallion using segments and sectors of circular discs

Procedure:
- Cut circular discs of same size.
- Fold the circle in half. Fold both layers as shown the angle at x being 30°.
- Fold both layers of the flap back as shown. Then unfold the folds and open the circle.
- Refold the folds so the motif looks like this. Make 6
- First layer Place over second layer
- Second layer medallion

**ACTIVITY-3**

Objective: To prepare 3-D decorative object from a 2-d circular disc.

Procedure: Cut Circular discs of same size. Then draw equilateral triangle on it. Fold each of the three segments formed as shown in picture. Join them as shown.
Use your imagination and create different objects.
WEB LINKS

- Formula of Area of a circle---- https://www.youtube.com/watch?v=YokKp3pwVFc
- Sector-https://www.youtube.com/watch?v=tD6wigYAYMk
- Area of a sector-https://www.youtube.com/watch?v=cAOVS2DU0U
- Areas related to figures and solids- https://www.youtube.com/watch?v=ZJ-VMbLTaU
CHAPTER-13

SURFACE AREA AND VOLUMES

INTRODUCTION

In our daily life we come across many combined solids like a cylinder over hemisphere, a capsule a cylinder attached to hemispheres at both the ends, a toy with a cone over hemisphere etc.

While calculating the surface area of such solids, we should calculate only the areas that are visible to our eyes. While calculating the volumes of such solids, we should calculate the volumes of all the solids involved.

A thorough understanding to visualize the different solids present in the combination of solids is required to solve the problems.

EXPECTED LEARNING OUTCOMES

1. To determine the surface area of an object formed by combining any two of the basic solids, namely, cuboid, cone, cylinder, sphere and hemisphere.
2. To find the volume of objects formed by combining any two of the following namely cuboid, cone, cylinder, sphere and hemisphere.
3. To understand that the volume of a solid remains same after its conversion into another shape.
4. To identify objects in the shape of frustum of a cone.
5. To find the surface area (curved and total) of a frustum of a cone.
6. To find the volume of a frustum of a cone.
GRADED EXERCISES

LEVEL -I

1. If two cubes of edge 3cm each are joined end to end, find the surface area of the resulting cuboid. (1m)

2. What will happen to the volume of the new shape during conversion of a solid from one shape to another? (1m)

3. Metallic spheres of radii 6cm, 8cm and 10cm respectively, are melted to form a single sphere. Find the radius of the resulting sphere. (2m)

4. If the total surface area of a solid hemisphere is 462cm$^2$, find its volume. (2m)

5. What is the height of a cone whose base area and volume are numerically equal? (3m)

6. The perimeters of the ends of a frustum of a cone are 36cm and 48cm. If the height of the frustum of a cone is 11cm, find its volume. (3m)

7. The largest sphere is carved out of a cube of side 7cm. Find the volume of the sphere (3m).

8. How many silver coins, 1.75cm in diameter and of thickness 2mm, must be melted to form a cuboid of dimensions 5.5cm x 10cm x 3.5cm? (4m)

LEVEL -2

1. A cone, a hemisphere and a cylinder stand on equal bases and have the same height. Show that their volumes are in the ratio 1: 2: 3. (3m)

2. How many spherical bullets can be made out of a solid cube of lead whose edge measures 44 cm, each bullet being 4 cm in diameter? (4m)

3. A solid metallic hemisphere of radius 8 cm is melted and recasted into a right circular cone of base radius 6 cm. Determine the height of the cone. (4m)

4. Water flows at the rate of 10m/minute through a cylindrical pipe of 5 mm in diameter. How long would it take to fill a conical vessel whose diameter at the base is 40 cm and depth 24 cm? (4m)

5. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π. (3m)

6. A solid iron cuboidal block of dimensions 4.4 m × 2.6 m × 1m is recast into a hollow cylindrical pipe of internal radius 30cm and thickness 5cm. Find the length of the pipe. (4m)

7. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid. (4m)

8. A cone, a hemisphere and a cylinder stand on equal bases and have the same height. Find the ratio of their surface areas. (4m)

9. A heap of rice is in the form of a cone of diameter 9 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap? (4m)

10. A solid wooden toy is in the shape of a right circular cone mounted on a hemisphere. If the radius of hemisphere is 4.2cm and the total height of the toy is 10.2cm, find the volume of the wooden toy. (4m)
1. 500 persons are taking a dip into a cuboidal pond which is 80 m long and 50 m broad. What is the rise of water level in the pond, if the average displacement of the water by a person is 0.04 m$^3$? (4m)

2. A factory manufactures 120000 pencils daily. The pencils are cylindrical in shape each of length 25 cm and circumference of base as 1.5 cm. Determine the cost of colouring the curved surfaces of the pencils manufactured in one day at Rs 0.05 per dm$^2$. (4m)

3. Solid spheres of diameter 6 cm are dropped into a cylindrical beaker containing some water and are fully submerged. If the diameter of the beaker is 18 cm and the water rises by 40 cm, find the number of solid spheres dropped in the water. (4m)

4. A cone is divided into two parts by drawing a plane through the midpoint of its axis, parallel to its base. Compare the volumes of the two parts. (3m)

5. The barrel of a fountain pen, cylindrical in shape, is 7 cm long and 5 mm in diameter. A full barrel of ink in the pen is used up on writing 3300 words on an average. How many words can be written in a bottle of ink containing one fifth of a litre? (3m)

6. An oil funnel made of tin sheet consists of a 10 cm long cylindrical portion attached to a frustum of a cone. If the total height is 22 cm, diameter of the cylindrical portion is 8 cm and the diameter of the top of the funnel is 18 cm, find the area of the tin sheet required to make the funnel (see the given figure). (4m)

7. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter 10cm of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid. (3m)

8. The radii of the internal and external surfaces of a metallic spherical shell are 3cm and 5cm respectively. It is melted and recast into a solid right circular cylinder of height 32/3 cm. Find the diameter of the base of the cylinder. (4m)

9. A cylindrical container of radius 6cm & height 15cm is filled with ice-cream. The whole ice cream has to be distributed to 10 children in equal cones with hemispherical tops. If the height of the conical portion is 4 times the radius of its base, find the radius of the cone. (4m)

10. An inverted cone of vertical height 12cm and radius of the base 9cm has water to a depth of 4cm. Find the area of the internal surface of the cone not in contact with water. (3m)

**VALUE BASED QUESTIONS**

1. Two types of water tankers are available in a shop. One is in a cubic form of dimensions 1 m x 1m x 1 m and another is in the form of cylindrical form of diameter 1 m and height is also 1 m. The shopkeeper advises to purchase cuboid tank to a customer.
   (i) Calculate the volume of the both tankers.
(ii) Which value is depicted by the shopkeeper?

2. Priyanshu wanted to make a bird-bath for his garden in the shape of a cylinder with a hemispherical depression at one end. The height of the cylinder is 1.4 metre and its radius is 42 cm.
   (i) Find the total surface area of the bird-bath.
   (ii) What value Priyanshu is depicting?

3. A farmer wants to dig a well either in the form of cuboid of dimensions (1m x 1m x 7m) or in the form of cylinder of diameter 1 meter and radius 7m. The rate to dig the well is Rs. 50/m³. Find the cost to dig both wells. The farmer decides to dig the cylindrical well. By his decision which value is depicted?

4. An ice cream seller has two types of ice cream container in the form of cylindrical shape and a cone with hemi-spherical base. Both have same height of 7 cm and same diameter of 7 cm. The cost of container are same but the seller decide to sell ice cream in cylindrical containers. (i) Calculate the volume of the both containers. (ii) Which value is depicted by the seller?

ERROR ANALYSIS

<table>
<thead>
<tr>
<th>S.No</th>
<th>POSSIBLE ERRORS</th>
<th>REMEDATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Many students take slant height as the vertical height and slant height $= \sqrt{h^2 + (r_1^2 - r_2^2)}$ instead of $\sqrt{h^2 + (r_1 - r_2)^2}$</td>
<td>Correct use of formula has to be emphasized, differentiate and drilled</td>
</tr>
<tr>
<td>2</td>
<td>Some students take diameter in place of radius and calculate</td>
<td>Difference between diameter and radius should be given practice while substituting in the formula</td>
</tr>
<tr>
<td>3</td>
<td>Some students do not identify whether to find the volume or surface area for the given information</td>
<td>Sufficient practice of questions of mensuration should be given where application of knowledge is involved. Calculate surface area-Use nets and solids and clear the ideas.</td>
</tr>
<tr>
<td>4</td>
<td>While finding TSA of a combined figure students find the sum of the TSA of two figures separately</td>
<td>Emphasize the common base area of combined figure should be excluded</td>
</tr>
</tbody>
</table>

QUESTION BANK

1. The sum of length, breadth and height of a cuboid is 19 cm and its diagonal is $5\sqrt{5}$ cm. What is its surface area?

2. A solid cylinder of radius $r$ and height $h$ is placed over other cylinder of same height and radius. Find the total surface area of the shape so formed.

3. A solid is hemispherical at the bottom and conical above. If the curved surface area of the two parts are equal, then find the ratio of the radius and height of the conical part.

4. The slant height of the frustum of a cone is 5 cm. If the difference between the radii of its two circular ends is 4 cm, find the height of the frustum.

5. A spherical bowl of internal diameter 36 cm contains a liquid. This liquid is to be filled in cylindrical bottles of radius 3 cm and height 6 cm. How many bottles are required to empty the bowl?
6. Three cubes of iron whose edges are in the ratio 3 : 4 : 5 are melted and converted into a single cube whose diagonal is \(12\sqrt{3}\) cm. Find the edges of the three cubes.

7. A right triangle whose sides are 15 cm and 20 cm is made to revolve about its Hypotenuse. Find the volume and surface area of the double cone so formed.

8. The lower portion of a hay stack is an inverted cone frustum and the upper part is a cone. Find the total volume of the hay stack.

9. A conical vessel of radius 6cm and height 8cm is completely filled with water. A sphere is lowered into the water and its size is such that when it touches the sides, it is just immersed as shown in the figure. What fraction of water flows out.
WEB LINKS

- Surface area - https://www.youtube.com/watch?v=s3KpK3hTD4
  https://www.youtube.com/watch?v=4khzn8j5d18

- Surface area of a cone - http://www.mathopenref.com/conearea.html
- Volume of sphere- https://www.youtube.com/watch?v=xuPl_80_j7k

http://www.mathopenref.com/spherearea.html

- Frustum of a cone - https://www.youtube.com/watch?v=ADH7VwBjOdc
  https://www.youtube.com/watch?v=0Ps7O01ed_Y

- Conversion of solids- https://www.youtube.com/watch?v=h-WIw2Q6JVM
CHAPTER-15

PROBABILITY

INTRODUCTION

"When you calculate the probability of an event you look at chances of getting what you want versus all the possible things that can happen. The probability of an event that you know for sure will happen is 100% or 1 while the probability of an event that will never happen is 0% or just plain 0."

What about other events that you're not so sure about? The probability of these events can be given as a percent or as an odds ratio. Let's pick something a little silly but simple as an example. Let's pretend that you want to wear a sweater to school and you have a blue sweater and a yellow sweater. The probability of wearing the blue sweater is 50% or the odds are 1 out of 2. What is the probability of wearing the yellow sweater? It would be the same. The probability of all the events that are possible must add up to 100%.

EXPECTED LEARNING OUTCOMES

1. To differentiate between experimental probability and theoretical probability
2. To understand the terms-experiment, random experiment, sample space, and an event.
3. To identify a sure event and an impossible event.
4. To find the sample space of a random experiment like tossing of coins, throwing a die/dice, drawing a card/cards from a pack of cards etc.
5. To find the probability of a given event using the theoretical probability formula
6. To find the sample space when Probability of the event and favorable outcomes are given.

CONCEPT MAP
GRADED EXERCISES

LEVEL -1

1. What is the sum of the probability of an event and non-event? (1m)
2. Write the sample space when two coins are tossed simultaneously. (2m)
3. If \( P(E) = 0.06 \), what is \( P(\text{not} E) \)? (1m)
4. A coin is tossed. Find the probability that a head is obtained. (1m)
5. If the probability of winning a game is \( \frac{1}{4} \), what is the probability of losing it? (1m)
6. In a single throw of a die, what is the probability of getting an even number? (2m)
7. In a single throw of a die, what is the probability of getting a perfect square? (2m)
8. A bag contains 5 black, 7 red and 3 white balls. A ball is drawn from the bag at random. Find the probability that the ball drawn is (i) red (ii) black or white (iii) not black (3m)
9. Find probability of throwing 5 with an ordinary dice. (1m)
10. Tickets numbered from 1 to 20 are mixed together and a ticket is drawn at random. What is the probability that the ticket has a number which is multiple of 3 or 7? (3m)

LEVEL -2

1. What are the total outcomes when we throw three coins simultaneously? (4m)
2. A bag contains 100 identical tokens, on which numbers 1 to 100 are marked. A token is drawn at random. What is the probability that the number on the token is: (a) an even number (b) an odd number (c) a multiple of 3 (d) a multiple of 5 (f) a multiple of 3 and 5 (h) a number less than 20? (4m)
3. A card is drawn from a well-shuffled pack of cards. Find the probability that the card drawn is: (a) a queen (b) a king bearing diamond sign (c) a black card (d) a jack (e) black and a queen (4m)
4. A letter is chosen at random from the letters of the word ‘ASSASSINATION’. Find the probability that the letter chosen is S? (2m)
5. If three coins are tossed simultaneously, then find the probability of getting at least two heads. (3m)
6. Two dice are thrown at the same time. Find the probability of getting different numbers on both dice. (3m)
7. Two unbiased coins are tossed simultaneously. Find the probability of getting (i) at least one head. (ii) at most one head. (iii) No head. (3m)
8. Find the probability that a leap year selected at random will contain 53 Sundays. (2m)
9. A card is drawn from a well-shuffled pack of cards. Find the probability that the card drawn is: (a) a queen (b) a king bearing diamond sign (c) a black card (d) a jack and a queen (4m)
10. The king, queen and jack of clubs are removed from a deck of 52 playing cards and the well shuffled. One card is selected from the remaining cards. Find the probability of getting: (i) a heart (ii) a king (iii) a club (iv) the ‘10’ of hearts. (4m)

LEVEL-3

1. A person is known to hit the target in 3 shots out of 4 shots. Find the probability that the target is not hit. (1m)
2. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears (i) a two digit number (ii) a perfect square (iii) a number divisible by 5. (3m)
3. A bag contains 5 red balls and some blue balls. If the probability of drawing blue ball is double that of a red ball, find the number of blue balls in the bag. (2m)
4. A box contains 12 balls out of which x are white. (i) If one ball is drawn at random, what is the probability that it will be a white ball? (ii) If 6 more white balls are put in the bag, the probability of drawing a white ball will be double than that in (i). Find x. (4m)
5. A game consists of tossing a one rupee coin three times and noting its outcome each time. Hanif wins if all the tosses give the same result i.e. three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game. (3m)
6. A jar contains 24 marbles some are green are others are blue. If a marble is drawn at random from the jar, the probability that it is green is 2/3. Find the number of blue marbles in the jar. (3m)

7. A coin is tossed successively three times. Find probability of getting exactly one head or two heads. (2m)

8. Three coins are tossed once. Find probability of: (a) 3 heads (b) exactly 2 heads (c) atleast 2 heads (d) atmost 2 heads (4m)

9. Two dice are thrown simultaneously. Find: (a) P(an odd number as a sum) (b) P(a doublet of odd numbers) (c) P(a total of atleast 9) (d) P( a multiple of 2 on one die and a multiple of 3 on other die) (4m)

10. Two black kings and two black jacks are removed from a pack of 52 cards. Find the probability of getting: (a) a card of hearts (b) a black card (c) either a red card or a king (3m)

VALUE BASED QUESTIONS

1. A carton consists of 100 shirts of which 88 are good, defect free shirts; 8 have minor defects; and 4 have major defects. Jimmy, a trader, will accept & sell the shirts which are good, defect free. But Sujatha, another trader, will only reject the shirts that have major defects & sell the acceptable lot at the same price. One shirt is drawn at random from the carton.
   What is the probability that
   (i) It is acceptable to Jimmy?
   (ii) Is acceptable to Sujatha?
   (iii)What values (any two) do you think distinguish Jimmy as a better human being?

2. A selection committee interviewed some people for the post of Sales Manager. The committee wanted that the female candidates should also be given the fairchance. So they called male and female candidates in 3:4 ratio.
   a. What is the probability of a female candidate being selected?
   b. Which value is shown by the selection committee?

3. 12 defective ball pens are accidentally mixed with 156 good one. It is not possible to just look at pen and tell whether or not it is defective. Theshopkeeper draws one pen at random.
   a. Determine the probability that the pen taken out is a good one.
   b. Suppose the pen drawn is defective. The shopkeeper did not sell out and kept the pen aside. He again draws one more pen at random from the rest. What is the probability that pen is not defective.
   c. Which value is shown by the shopkeeper?
4. In a survey, it was found that 40% people use petrol, 35% use diesel and remaining use CNG for their vehicles.

(a) Find the probability that a person chosen at random uses CNG.
(b) Which fuel out of the above three is appropriate for the welfare of the society?

**ERROR ANALYSIS**

<table>
<thead>
<tr>
<th>SI no</th>
<th>POSSIBLE ERRORS</th>
<th>REMEDIATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Some students do not identify total number of samples but only guess</td>
<td>Sample space should be taught clearly using activity. Concept of total outcomes and favourable outcomes should be made clear.</td>
</tr>
<tr>
<td>2</td>
<td>Some students add 2 joker cards to total number of cards in a pack</td>
<td>Students should be shown a pack of cards and remove the joker and emphasize there are 52 cards only</td>
</tr>
<tr>
<td>3</td>
<td>While a random experiment of tossing three coins are done student get confused in identifying the total no of trials</td>
<td>Activity should be repeated many times till students remember total number of trials</td>
</tr>
</tbody>
</table>
1. In the adjoining figure a dart is thrown at the dart board and lands in the interior of the circle. What is the probability that the dart will land in the shaded region.

2. In the adjoining figure ABCD is a square with sides of length 6 units points P & Q are the mid points of the sides BC & CD respectively. If a point is selected at random from the interior of the square what is the probability that the point will be chosen from the interior of the triangle APQ.

3. In a musical chair game the person playing the music has been advised to stop playing the music at any time within 2 minutes after she starts playing. What is the probability that the music will stop within the half minute after starting.

4. In the fig points A, B, C and D are the centres of four circles, each having a radius of 1 unit. If a point is chosen at random from the interior of a square ABCD, what is the probability that the point will be chosen from the shaded region.

5. A number x is selected from the numbers 1, 2, 3 and then a second number y is randomly selected from the numbers 1, 4, 9. What is the probability that the product xy of the two numbers will be less than 9?
6. A jar contains 54 marbles each of which is blue, green or white. The probability of selecting a blue marble at random from the jar is 31 and the probability of selecting a green marble at random is 94. How many white marbles does the jar contain?

7. An integer is chosen at random from the first two hundreds digit. What is the probability that the integer chosen is divisible by 6 or 8.

8. Find the probability that the month June may have 5 Mondays in
   (i) a leap year  
   (ii) a non-leap year

9. A child’s game has 8 triangles of which 3 are blue and rest are red and 10 squares of which 6 are blue and rest are red. One piece is lost at random. Find the probability that it is a: (i) triangle (ii) square (iii) square of blue of colour (iv) triangle of red colour.

10. A die has it 6 faces marked 0, 1, 1, 1, 4, 4. Two such dice are thrown together and the total sum is recorded. How many different sums are possible?

**ACTIVITIES/PROJECTS**

**ACTIVITY-1**

Rock, Paper, Scissors - The Study of Chance

Objective- To introduce basic information on probability and statistics. It can be used as an introduction to a unit on probability. It should be followed up with a discussion about how probability is used in the real world. This activity can be made as simple or as complex as necessary depending on grade level.

Materials:
- Two sets of hands
- Paper
- Pencil

After this activity, the student will be able to determine whether or not the game is fair and be able to interpret and display the data obtained. The student will also be able to see that probability is used often in society.

Procedures:
Divide the class into pairs and have them play the game eighteen times. A rock is a closed Fist. Paper is palm on palm, and scissors is the number two horizontally. The student hits their other hand twice, and on the third time gives the symbol they wish. A rock beats scissors. Paper beats rocks, and scissors beats paper. Instruct the students to keep a record of wins and losses.

Once the class has finished, record the results for player A is one color, and player B in another color. Then, the students can figure mean, mode, and range each set of data.

Now draw a tree diagram to show all possible outcomes.

Answer the following questions to determine if the game is fair.

1. How many outcomes does the game have? (9)
2. Label each possible outcome on the tree diagram as to win for a, b, or tie.
3. Count the number of wins for A. (3)
4. Find the probability A will win in any round. (3/9 = 1/3)
5. Count the number of wins for B. (3)
6. Find the probability B will win in any round. \( \frac{3}{9} = \frac{1}{3} \)
7. Is the game fair? Do both players have an equal probability of winning any round? (yes)
8. Compare the mathematical model with what happened when the students played the game.
9. How do you think probability is used in the real world? See how many areas you can list that use probability.

**ACTIVITY-2**

**Objective:** – To appreciate that finding probability through experiment is different from finding probability by calculation. Students become sensitive towards the fact that if they increase the number of observations, probability found through experiment approaches the calculated probability.

(i) The student work individually or at most in groups by performing the basic experiments like tossing of coins, throwing a die etc.

**ACTIVITY-3**

**Objective:** To find the probability of an outcome and compare it with its theoretical probability.

**Materials Required:** Geometry box, thick white card sheet, sketch pens.

**Procedure:**

1. On a piece of thick card sheet, draw a circle of radius 5 cm. Now using your compasses, divide the circle into 8 equal parts by marking off angles of 45° at the centre of the circle. Mark points A, B, C, ..., H on the circle and join AB, BC, CD, ..., GH to get a regular octagon.

2. Cut off the octagon ABCDEFGH. Insert a small piece of pencil stub to make a top as shown in figure 1. Number the triangles 1, 2, 3, ..., 8.
3. Spin the top 40 times and record in table 1, the frequency with which each of the numbers 1 to 8 touches the table or floor.
4. Calculate the experimental probability obtaining each number.
5. Calculate the theoretical probability of obtaining each score.
6. Draw a combined bar graph to compare the probabilities obtained in steps 4 and 5 above. Write your observations.
### Table 1

<table>
<thead>
<tr>
<th>Number</th>
<th>Tally-marks</th>
<th>Frequency</th>
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<tr>
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<tr>
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<td>/III</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>III/III</td>
<td>8</td>
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<td>4</td>
<td>//III</td>
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<tr>
<td>5</td>
<td>III III</td>
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<tr>
<td>6</td>
<td>II</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>III</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>III</td>
<td>5</td>
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<tr>
<td><strong>Total</strong></td>
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</table>

### Table 2

<table>
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<tr>
<th>Number</th>
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<th>Theoretical Probability</th>
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<tbody>
<tr>
<td>1</td>
<td>$\frac{3}{40}$</td>
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<tr>
<td>2</td>
<td>$\frac{4}{40}$</td>
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<td>3</td>
<td>$\frac{8}{40}$</td>
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<tr>
<td>8</td>
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</tr>
</tbody>
</table>
Observations: The experimental and theoretical probabilities are equal in three out of eight cases as seen in table 2 and figure 2. Thus, there is a wide difference between the experimental and theoretical probability of an event in the activity performed above.

POWER POINT PRESENTATIONS

WEBLINKS

- https://www.youtube.com/watch?v=GH0BWluXxtM
- https://www.youtube.com/watch?v=usY1oY0bWQ8
- https://www.youtube.com/watch?v=rIUZXRtJGu8
Q1. Fill in the blanks:
(a) The number of hours from 8 p.m. Tuesday until 5 am Friday of the same week is ……………
(b) If $3 \times x - 2 = 81$ then x equals………….
(c) In a school the ratio of boys to girls is 3:5 and the ratio of girls to teachers is 6:1. The ratio of boys to teachers is ………….
(d) If $7n + 9 > 100$ and n is an integer the smallest possible value of n is ………
(e) In the diagram, AC = 4, BC = 3, and BD = 10. The area of the shaded triangle is ………….

Solution:
1. (a) Tuesday 8 pm to Thursday 8 pm = 24 * 2 = 48 hrs
   Thursday 8 pm to Friday 5 am = 9 hrs
   Total hours = 57 hrs

   (b) $3^{x - 2} = 81$

   $\Rightarrow 3^{x - 2} = 3^4$

   $\Rightarrow x - 2 = 4$

   $\Rightarrow x = 6$

   (c) Boys: Girls = 3:5 = 18:30
       Girls: Teacher = 6:1 = 30:5
       Boys: Teacher = 18:5

   (d) $7n + 9 > 100$

   $\Rightarrow 7n > 91$

   $\Rightarrow n > 13$

   Smallest value of n=14

   (f) Required Area = $\frac{1}{2} \times CD \times AC$ sq. units
\[ \frac{1}{2} \times 7 \times 4 \text{ sq. units} = 14 \text{ sq. units} \]

Q2. (a) Find the number of positive integers less than or equal to 300 that are multiples of 3 or 5, but are not multiples of 10 or 15.

(b) The product of the digits of each of the three – digit numbers 138, 262 and 432 is 24. Write down all three digit numbers having 24 as the product of the digits.

Solution:

2. (a) No. of multiples of 3 \[= \left\lfloor \frac{300}{3} \right\rfloor \] = 100
No. of multiples of 5 \[= \left\lfloor \frac{300}{5} \right\rfloor \] = 60
No. of multiples of 3 and 5 both \[= \left\lfloor \frac{300}{15} \right\rfloor \] = 20
No. of multiples of 10 \[= \left\lfloor \frac{300}{10} \right\rfloor \] = 30
No of multiples of 15 \[= \left\lfloor \frac{300}{15} \right\rfloor \] = 20
No of multiples of 10nd 15both \[= \left\lfloor \frac{300}{30} \right\rfloor \] = 10

\(\left\lfloor x \right\rfloor\) denotes greatest integer less than equal x

Therefore,

Required number of numbers = (100 + 60 - 20) - (30 + 20 - 10)
= 140 - 40
= 100

(b) 24 can be written as a product of three numerals as -
1 \times 3 \times 8
1 \times 6 \times 4
2 \times 4 \times 3
2 \times 6 \times 2

For three different numerals there are 6 arrangements of each possible product and for fourth product having 2 two’s number of arrangements will be 6/2 = 3

All three digit numbers having product of their digits 24 =
138, 183, 318, 381, 813, 831,
164, 146, 461, 416, 614, 641,
243, 234, 342, 324, 432, 423,
262, 226, 622
Q3. (a) Solve:
\[ x^2 + xy + y^2 = 19 \]
\[ x^2 - xy + y^2 = 49 \]

(b) The quadratic polynomial \( p(x) = a(x - 3)^2 + bx + 1 \) and \( q(x) = 2x^2 + c(x - 2) + 13 \) are equal for all values of \( x \). Find the value of \( a, b, c \).

Solution:

3. (a) \[ x^2 - xy + y^2 = 49 \]
\[ x^2 + xy + y^2 = 19 \]

Adding
\[ 2(x^2 + y^2) = 68 \]
\[ \Rightarrow x^2 + y^2 = 34 \]

Subtracting
\[ 2xy = -30 \]
Hence, \((x - y)^2 = 64\)
\[ x - y = \pm 8 \]
\[ (x + y)^2 = 4 \]
\[ x + y = \pm 2 \]

Solving equations possible values of \( x \) and \( y \) are:
\[ x = 5, \quad y = -3 \]
\[ x = -5, \quad y = 3 \]
\[ x = 3, \quad y = -5 \]
\[ x = -3, \quad y = 5 \]

(b) \( p(x) = a(x - 3)^2 + bx + 1 \)
\( q(x) = 2x^2 + c(x - 2) + 13 \)
\( P(x) = q(x) \)
\[ \Rightarrow a(x^2 - 6x + 9) + bx + 1 = 2x^2 + cx - 2c + 13 \]
\[ \Rightarrow a(x^2 - 6x + 9) + bx + 1 = 2x^2 + cx - 2c + 13 \]
\[ \Rightarrow (a - 2)x^2 + (-6a + b - c)x + 9a + 2c - 12 = 0 \]
\[ a - 2 = 0, \quad -6a + b - c = 0, \quad 9a + 2c - 12 = 0 \]
\[ \Rightarrow a = 2, \quad b = 9 \quad \text{and} \quad c = -3 \]

Alternate method:
\[ x = 3 \Rightarrow 3b + 1 = 18 + c + 13 \Rightarrow 3b - c = 30 \]
\[ x = 0 \Rightarrow 9a + 1 = -2c + 13 \Rightarrow 9a + 2c = 12 \]
\[ x = 2 \Rightarrow a + 2b + 1 = 8 + 13 \Rightarrow a + 2b = 20 \]
Solving equations: \( a=2, b=9 \) and \( c=-3 \)

Q4.(a) Two squares, each with side length 5 cm, overlap as shown. The shape of their overlap is a square, which has an area of 4 cm\(^2\). Find the perimeter, in centimeters, of the shaded figure.

(b) A rectangle is divided into four smaller rectangles. The areas of three of these rectangles are 6, 15 and 25, as shown. Find the area of the shaded rectangle.

Solution: 4. (a)

From the figure:
Perimeter of shaded portion \( = (4 \times 5 + 4 \times 3) = 32 \) cm.

4. (b)
From figure areas of given regions

\[ xz = 6 \]
\[ yz = 15 \]

\[ \frac{x}{y} = \frac{2}{5} \]
\[ \Rightarrow y = \frac{5}{2} x \]
\[ u \times \frac{5}{2} x = 25 \]
\[ xu = \frac{25 \times 5}{2} = 10 \]

Hence,

Area of fourth rectangle = 10 sq. units

Q5. (a) A square ABCD is inscribed in a circle of unit radius. Semicircles are described on each side as a diameter. Find the area of the region bounded by the four semi-circles and the circle.
(b) In a parallelogram ABCD, H is the mid-point of AB and M is the mid-point of CD. Show that AM and CH divide the diagonal DB in three equal parts.

Solution: 5. (a)

Diagonal of square = 2 units

Side of square = \( \sqrt{2} \) units

Area of four semicircles

\[ \Rightarrow 4 \times \frac{1}{2} \pi \times \left( \frac{\sqrt{2}}{2} \right)^2 \text{ sq. units} \]

\[ = 4 \times \frac{1}{2} \pi \times \left( \frac{2}{4} \right) \text{ sq. units} \]

\[ = \pi \text{ sq. units} \]

Therefore,

Area of four segments = \( (\pi - 2) \) sq. units
Area of required region = \{\pi - (\pi - 2)\} sq. units
\[= 2 \text{ sq. units}.
\]

(b) \[\text{AH} = \text{CM}\]
\[\text{AH} \parallel \text{CM}\]
Therefore,
Quadrilateral AMCH is a parallelogram
In triangle ABP, H is mid point
\[\text{HQ} \parallel \text{AP}\]
Therefore,
\[\text{Q} \text{ is mid point of BP} \implies \text{BQ} = \text{PQ}\]
Similarly,
\[\text{In triangle DCQ} \implies \text{DP} = \text{PQ}\]
\[\implies \text{BQ} = \text{PQ} = \text{DP}\]

Q6. A two-digit number has the property that the square of its tens digit plus ten times its units digit is equal to the square of its units digit plus ten times its tens digit. Find all two digit numbers which have this property, and are prime numbers.

Solution:
6. Let,
Ten’s digit = x and one’s digit = y
\[x^2 + 10y = y^2 + 10x\]
\[\implies x^2 - y^2 = 10(x - y)\]
\[\implies (x - y)(x + y - 10) = 0\]
Either \[x - y = 0\] or \[x + y = 10\]
\[x = y\]
Only 11 is such prime number
For \[x + y = 10\]
Numbers may be 19, 28, 37, 46, 55, 64, 73, 82, 91
All two digit prime numbers having the property = 11, 19, 37, 73

Q7. In the diagram, it is possible to travel only in the direction indicated by the arrow. How many different routes from A to B are there in all?
Solution: Number of routes through AXB = 2 \times 1
Number of routes through AXYB = 2 \times 1 \times 3
Number of routes through AYB = 1 \times 3
Total number of routes = 2 \times 1 + 2 \times 1 \times 3 + 1 \times 3
= 11

Q8. The Object shown in the diagram is made by gluing together the adjacent faces of six wooden cubes, each having edges of length 2 cm. Find the total surface area of the object in square centimeters.

Solution:
Surface area of object = Number of visible faces after gluing \times area of one face
Total surface area of object = (5+4+4+4+5) \times 2^2 \text{ cm}^2
= 104 \text{ cm}^2

Or
Faces visible after gluing = Total faces before gluing – faces glued
= 6 \times 6 - 2 \times 5
= 26
Surface area = 26 \times 4 \text{ cm}^2
= 104 \text{ cm}^2

Q9. Six points A, B, C, O, E, and F are placed on a square grid, as shown. How many triangles that are not right-angled can be drawn by using 3 of these 6 points as vertices.
Solution:

Number of triangles that are not right triangle

\[
\frac{6 \times 5 \times 4}{1 \times 2 \times 3} - 1 - 1 - 12 = 20 - 14 = 6
\]
i.e. \{\Delta ABE, \Delta ACE, \Delta BCD, \Delta DEC, \Delta DFB, \Delta EFA\}

Or

For making triangles that are not right triangle we must select two points from one row and the third from the other which is not directly opposite to these two.

It can be done in \(2 \times (3 \times 1) = 6\) ways

Q 10. A distance of 200 km is to be covered by car in less than 10 hours. Yash does it in two parts. He first drives for 150 km at an average speed of 36 km/hr, without stopping. After taking rest for 30 minutes, he starts again and covers the remaining distance non-stop. His average for the entire journey (including the period of rest) exceeds that for the second part by 5 km/hr. Find the speed at which he covers the second part.

Solution:

\[\text{Let, Speed in II part} = x \text{ km/hr}\]
\[\text{Average speed} = (x + 5) \text{ km/h}\]
\[\text{Total time taken} = \left(\frac{150}{36} + \frac{1}{2} + \frac{50}{6}\right)\]
\[
\frac{150 + 14x}{3x} = \frac{200 \times 3x}{150 + 14x} = x + 5
\]

\[
7x^2 - 190x + 375 = 0 \\
(x - 25)(7x - 15) = 0 \\
x = 25 \text{ or } x = 15/7
\]

But \( \frac{50}{x} + \frac{14}{3} < 10 \), \( x = 15/7 \) does not satisfy it.

Therefore, \( x = 25 \)

Speed for next part = 25 km/hr

**11TH KVS MATHS OLYMPIAD CONTEST-2008**

Questions with solution:

1. Find the value of \( S = 1^2 - 2^2 + 3^2 - 4^2 + \ldots - 98^2 + 99^2 \).

Remember:

(i) Any odd number can be written in the form \( 2n-1 \) or \( 2n + 1 \)

Any even number can be written in the form \( 2n \).

(ii) Sum of the squares of first \( N \) natural numbers is \( \frac{N(N+1)(2N+1)}{6} \).

(iii) This can also be expressed using the \( \Sigma \)-notation as:

\[
\sum_{n=1}^{N} n^2 = 1^2 + 2^2 + 3^2 + \ldots + N^2 = \frac{N(N+1)(2N+1)}{6}
\]

Solution:
Alternate method:

\[ S = 1^2 - 2^2 + 3^2 - 4^2 + \ldots \ldots - 98^2 + 99^2. \]

\[ = (1^2 - 2^2) + (3^2 - 4^2) + \ldots + (97^2 - 98^2) + 99^2. \]

\[ = \sum_{n=1}^{49} (2n-1)^2 - (2n)^2 + 99^2 \quad \text{(first no in each bracket is odd, 2nd no is even)} \]

\[ = \sum_{n=1}^{49} -4n + 1 + 99^2 \quad \text{((2n-1)^2 - (2n)^2 is simplified)} \]

\[ = -\sum_{n=1}^{49} 4n - 1 + 99^2 \]

\[ = 99^2 - \sum_{n=1}^{49} 4n - 1 \]

\[ = 99^2 - \left\{ \frac{4 \times 49 \times 50}{2} - 49 \right\} \]

\[ = 99^2 - 49 \times 99 \]

\[ = 99 \times 50 \]

\[ = 4950. \]

Prime factors of 15 are 3, 5.

Therefore any multiple of 15 must be divisible by 3 and 5.
As the required no has to be divisible by 5, it should end in zero
(the option 5 is not applicable here)
Also, the given no must be divisible by 3.
Therefore if you put one 8 or two eights or one 8 and zero before zero
i.e. 80 or 880 or 800 or 8080 are not divisible by 3.
Also, we want the smallest multiple of 15 and therefore the only possibility is 8880.
The required no is 8880.

3. At the end of year 2002, Ram was half as old as his grandfather. The sum of years in which they were born is 3854. What is the age of Ram at the end of year 2003?
Let grandfather's age at the end of 2002 be ‘x’ years
Therefore Ram’s age at the end of 2002 will be x/2 years.
Accordingly, the year in which they were born will be (2002 - x), (2002 - x/2)
(2002 - x) + (2002 - x/2) = 3854.
Solving this simple eqn gives x = 100.
Therefore age of Ram at the end of 2002 will be 50 and his age at the end of 2003 will be 51 years.

4. Find the area of the largest square, which can be inscribed in a right triangle with legs ‘4’ and ‘8’ units.
5. In a triangle the length of an altitude is 4 unit and this altitude divides the opposite side in two parts in the ratio 1:8. Find the length of a segment parallel to altitude which bisects the area of the given triangle.

\[ AB = 8, \ BC = 4, \therefore AC^2 = 64 + 16 = 80 \Rightarrow AC = \sqrt{80} \]

Let each side of the square be 'x' units and EC = 'y' units.

\[ \text{DE} \parallel \text{BC} \Rightarrow \frac{\text{AD}}{\text{DB}} = \frac{\text{AE}}{\text{EC}} \Rightarrow \frac{8 - x}{y} = \frac{\sqrt{80} - y}{x} \quad \text{(i)} \]

\[ \text{EG} \parallel \text{AB} \Rightarrow \frac{x}{4 - x} = \frac{\sqrt{80} - y}{y} \quad \text{(ii)} \]

\[ (i) \& (ii) \Rightarrow \frac{8 - x}{x} = \frac{x}{4 - x} \]

Crossmultiplying and solving gives \( x = \frac{8}{3} \)

\[ \therefore \text{Area of the square} = \frac{64}{9} \text{ sq.units} \]

5. In a triangle the length of an altitude is 4 unit and this altitude divides the opposite side in two parts in the ratio 1:8. Find the length of a segment parallel to altitude which bisects the area of the given triangle.

\[ \begin{align*}
\text{AD} &= 4, \quad \text{EF} \parallel \text{AD} \quad \text{and divides the triangle ABC into two equal parts.} \\
i.e. \quad \text{ar} \triangle \text{CEF} &= \frac{1}{2} \text{ar} \triangle \text{ABC}. \\
\text{BD} : \text{DC} &= 1:8 \quad \text{(given)} \\
\therefore \text{Let BD} &= x, \quad \text{DC} = 8x, \quad \text{EF} = y. \\
\text{We have to find EF, i.e. } y. \\
\text{EF} \parallel \text{AD} \Rightarrow \triangle \text{CEF} \equiv \triangle \text{CAD} \Rightarrow \frac{\text{EF}}{\text{AD}} = \frac{\text{CF}}{\text{CD}} \Rightarrow \frac{y}{4} = \frac{\text{CF}}{8x} \Rightarrow \text{CF} = 2xy \\
\text{Now, } \ar \triangle \text{CEF} = \frac{1}{2} \ar \triangle \text{ABC.} \Rightarrow \frac{1}{2} \times \text{EF} \times \text{CF} = \frac{1}{2} \times \frac{1}{2} \times \text{BC} \times \text{AD} \\
\Rightarrow y \times 2xy = \frac{1}{2} \times 9x \times 4 \\
\Rightarrow y^2 = 9 \\
\Rightarrow y = 3 \text{ units.} \\
\end{align*} \]

6. A number ‘X’ leaves the same remainder while dividing 5814, 5430, 5958. What is the largest possible value of ‘X’.

[Diagram of triangle ABC with AD parallel to EF, dividing the triangle into equal parts.]
According to the given condition,

\[ 5814 = aX + r, \quad 5430 = bX + r, \quad 5958 = cX + r \]

and this implies the difference of any of the above 3 numbers is divisible by \( X \).

\[ 5814 - 5430 = 384, \quad 5958 - 5430 = 528, \quad 5958 - 5814 = 144. \]

The required number is H.C.F of 384, 528, 144.

Find the H.C.F as 48.

The required number here is 48.

Note that it is enough even if you find the H.C.F of any 2 of the numbers 384, 528, 144.

7. A sports meet was organized for 4 days. On each day, half of existing total medals and one more medal was awarded. Find the number of medals awarded on each day.

Let the total medals be \( x \).

No of medals awarded on 1st day = \( \frac{x}{2} + 1 \) ............(i)

Remaining medals = \( x - \left( \frac{x}{2} + 1 \right) = \frac{x}{2} - 1 \)

No of medals awarded on 2nd day = \( \frac{1}{2} \left( \frac{x}{2} - 1 \right) + 1 = \frac{x}{4} + \frac{1}{2} \) ............(ii)

Remaining medals = \( \frac{x}{2} - 1 - \left( \frac{x}{4} + \frac{1}{2} \right) = \frac{x}{4} - \frac{3}{2} \)

No of medals awarded on 3rd day = \( \frac{1}{2} \left( \frac{x}{4} - \frac{3}{2} \right) + 1 = \frac{x}{8} + \frac{1}{4} \) ............(iii)

Remaining medals = \( \left( \frac{x}{4} - \frac{3}{2} \right) - \left( \frac{x}{8} + \frac{1}{4} \right) = \frac{x}{8} - \frac{7}{4} \)

No of medals awarded on 4th day = \( \frac{1}{2} \left( \frac{x}{8} - \frac{7}{4} \right) + 1 = \frac{x}{16} + \frac{1}{8} \) ............(iv)

Now, \( (i) + (ii) + (iii) + (iv) = x \)

\( i.e., \quad \frac{x}{2} + 1 + \frac{x}{4} + \frac{1}{2} + \frac{x}{8} + \frac{1}{4} + \frac{x}{16} + \frac{1}{8} = x \)

Solve the above eqn and get the value of \( x \) as 30.
8. Let \( \triangle ABC \) be isosceles with \( \angle ABC = \angle ACB = 78^\circ \). Let \( D \) and \( E \) be the points on sides \( AB \) and \( AC \) respectively such that \( \angle BCD = 24^\circ \) and \( \angle CBE = 51^\circ \). Find \( \angle BED \) and justify your result.

From \( \triangle BEC \), \( \angle BEC = 51^\circ \Rightarrow BC = CE \) \( \ldots \)(i)

From \( \triangle BDC \), \( \angle BDC = 78^\circ \Rightarrow BC = CD \) \( \ldots \)(ii)

(i) & (ii) \( \Rightarrow BC = CD = CE \)

\( \therefore \) In \( \triangle CDE \), if \( \angle CDE = x = \angle CED \) then \( \angle BED = \angle CED - \angle CEB = x - 51^\circ \)

Now, from \( \triangle CED \), \( x + x + 54 = 180 \Rightarrow x = 63 \).

\( \therefore \angle BED = x - 51^\circ = 12^\circ \)

9. If \( \alpha, \beta, \) and \( \gamma \) are the roots of the equation \((x - a)(x - b)(x - c) + 1 = 0\), then show that \( a, b, \) and \( c \) are the roots of the equation 

\[(\alpha - x)(\beta - x)(\gamma - x) + 1 = 0.\]

Let \( p(x) = (x - a)(x - b)(x - c) + 1, \ q(x) = (\alpha - x)(\beta - x)(\gamma - x) + 1 \)

Since \( \alpha, \beta, \) and \( \gamma \) are the roots of \( p(x) \) we can write \( p(x) = (x - \alpha)(x - \beta)(x - \gamma) \).

Now, \( q(x) = (\alpha - x)(\beta - x)(\gamma - x) + 1 = -(x - \alpha)(x - \beta)(x - \gamma) + 1 = 1 - p(x) \)

\( q(x) = 1 - p(x) = 1 - [(x - a)(x - b)(x - c) + 1] = -(x - a)(x - b)(x - c) \Rightarrow \) roots of \( q(x) \) are \( a, b, c. \)
10. A 4 X 4 wooden cube is painted so that one pair of opposite faces is blue, one pair green and one pair red. The cube is now sliced into 64 cubes of side 1 unit each.

(i) How many of the smaller cubes have no painted face?
(ii) How many of the smaller cubes have one painted face?
(iii) How many of the smaller cubes have exactly two painted faces?
(iv) How many of the smaller cubes have exactly three painted faces?
(v) How many of the smaller cubes have exactly one face painted blue and
(vi) One face painted green?

For answer go through page -1 of the same project or the following link:


**JUNIOR MATHEMATICAL OLYMPIAD  2003**

Q 1. Fill in the blanks-
   (a) The digits of the number 2978 are arranged first in descending order and then in ascending order. The difference between the resulting two numbers is….
   (b) Yash is riding his bicycle at a constant speed of 12 kilometres per hour. The number of metres he travels each minute is ….
   (c) The square root of 35 X 65 X 91 is ….
   (d) The number 81 is 15% of ….
   (e) A train leaves New Delhi at 9.45 am and reaches Agra at 12.58 pm. The time taken in the journey, in minutes, is ….

**Ans:**
(a) Required difference = 9872 - 2789 = 7083
(b) No. of metres traveled in each minute = 12000/60 = 200 metres
(c) \(\sqrt{35 \times 65 \times 91} = 5 \times 7 \times 13 = 845\)
(d) Number = \(\frac{81 \times 100}{15} = 540\)
(e) Time taken in journey = 3 hr 13 minutes = 193 minutes

Q 2.
(a) Find the largest prime factor of 203203.
(b) Find the last two (ten’s and unit’s) digit of \((2003)^{2003}\).

**Ans:**
(a) 203203 = 7 \times 7 \times 11 \times 13 \times 29
Therefore, Largest prime factor = 29
(b) Last two digits is remainder when number is divided by 100
\[(2003)^2 \equiv 3^2 \pmod{100} \equiv 9 \pmod{100}\]
\[(2003)^4 \equiv 9^2 \pmod{100} \equiv -19 \pmod{100}\]
(2003)^8 \equiv (-19)^2 (\text{mod } 100) \equiv 61 (\text{mod } 100)

(2003)^{16} \equiv 61^2 (\text{mod } 100) \equiv 21 (\text{mod } 100)

(2003)^{32} \equiv 21^2 (\text{mod } 100) \equiv 41 (\text{mod } 100)

(2003)^{40} \equiv (2003)^{32} \cdot (2003)^8 (\text{mod } 100) \equiv 41.61 (\text{mod } 100) \equiv 1 (\text{mod } 100)

(2003)^{2000} \equiv (2003)^{40} \cdot 50 \equiv 1 \cdot 50 (\text{mod } 100) \equiv 1 (\text{mod } 100)

(2003)^{2003} \equiv 2003^{2000} \cdot 2003^2 \cdot 2003 (\text{mod } 100) \equiv 1.9.3 (\text{mod } 100) \equiv 27 (\text{mod } 100)

Last two digits of 2003^{2003} = 27

[Go through lesson on Arithmetic Congruencies by MsRajeshwari in the project for learning basic concepts of congruencies]

Q 3. (a) Find the number of perfect cubes between 1 and 1000009 which are exactly divisible by 9.

(b) If \( x = 5 + 2\sqrt{6} \), find the value of

(i) \( \sqrt{x} + \frac{1}{\sqrt{x}} \)

(ii) \( x^3 + \frac{1}{x^3} \)

Ans: (a) Perfect cubes divisible by 9 will be cubes of multiples of 3.

Since, \( 1 < x^3 < 1000009 \)

\[ 1 < x < 101 \]

Also \( x \) is a multiple of 3

But,

\( 101 = 3 \times 33 + 2 \)

Between 1 and 101 there are 33 multiples of 3.

Required number of perfect cubes = 33

(b)

\( x = 5 + 2\sqrt{6} \), \( \frac{1}{x} = 5 - 2\sqrt{6} \)

Therefore,

\[ \left( x + \frac{1}{x} \right) = 10 \]

\[ \sqrt{x} = \sqrt{3} + \sqrt{2} \]

\[ \frac{1}{\sqrt{x}} = \sqrt{3} - \sqrt{2} \]
Therefore,
\[ \sqrt{x} + \frac{1}{\sqrt{x}} = 2\sqrt{3} \]

And,
\[ x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) \]
\[ = 10^3 - 3 \times 10 \]
\[ = 970 \]

Q 4. (a) Solve:
\[ \frac{x^2 - 1}{x^2 - 4} \frac{x^2 - 5}{x^2 - 8} = \frac{x^2 - 2}{x^2 - 5} \frac{x^2 - 6}{x^2 - 9} \]
\[ \Rightarrow \frac{x^4 - 10x^2 + 9 + x^4 - 10x^2 + 24}{x^4 - 13x^2 + 36} = \frac{x^4 - 10x^2 + 16 + x^4 - 10x^2 + 25}{x^4 - 13x^2 + 40} \]
\[ \Rightarrow 24x^2 = 156 \]
\[ \therefore x = \pm \frac{\sqrt{13}}{2} \]

(b) Find the remainder when \( x^{81} + x^{49} + x^{25} + x^9 + x \) is divided by \( x^3 - x \).

Ans: (a)
\[ \frac{x^2 - 1}{x^2 - 4} \frac{x^2 - 5}{x^2 - 8} = \frac{x^2 - 2}{x^2 - 5} \frac{x^2 - 6}{x^2 - 9} \]
\[ \Rightarrow \frac{x^4 - 10x^2 + 9 + x^4 - 10x^2 + 24}{x^4 - 13x^2 + 36} = \frac{x^4 - 10x^2 + 16 + x^4 - 10x^2 + 25}{x^4 - 13x^2 + 40} \]
\[ \Rightarrow 24x^2 = 156 \]
\[ \therefore x = \pm \frac{\sqrt{13}}{2} \]

When \( x^{81} + x^{49} + x^{25} + x^9 + x \) is divided by \( x^3 - x \) remainder will be same as when \( x^{30} + x^{48} + x^{24} + x^8 + 1 \) is divided by \( x^2 - 1 \).

Taking \( y = x^2 \)
\[ x^{80} + x^{48} + x^{24} + x^8 + 1 = y^{40} + y^{24} + y^{12} + y^4 + 1 \]
\[ \therefore \text{Remainder} = 1^{40} + 1^{24} + 1^{12} + 1^4 + 1 = 5 \]
5. (a) OPQ is a quadrant of a circle and semicircles are drawn on OP and OQ. Areas a and b are shaded. Find a/b.

\[ \text{Area of region which is not shaded} = \text{area of quadrant} - (a + b) = \text{area of two semicircles} - 2a \]

\[ \Rightarrow \frac{\pi R^2}{4} - a - b = \frac{\pi \left(\frac{R}{2}\right)^2}{2} - 2a \]

\[ \Rightarrow a = b \]

(b) Assuming all vertical lines are parallel, all angles are right angles and all the horizontal lines are equally spaced, what fraction of the figure is shaded?
\[ \frac{a}{b} = 1 \]

(b)
Shaded area on horizontal shifting to one row makes one complete row.
\[ \therefore \text{Area of shaded portion} = \frac{1}{4} \text{ of total area of rectangle} \]

Q6. Alternate vertices of a regular hexagon are joined as shown. What fraction of the total area of the hexagon is shaded? (Justify your answer)

Ans: Let side of given regular hexagon = \(a\)

Base of isosceles triangle formed by sides of hexagon and line segment joining the alternate vertices = \(\sqrt{3} a\)

Side of equilateral triangles formed = one third of diagonals = \(\frac{\sqrt{3}}{3} a\)

Shaded area is forming regular hexagon of side \(\frac{\sqrt{3}}{3} a\)

\[ \therefore \text{Ratio of shaded area to the total area} = \frac{6 \times \frac{\sqrt{3}}{4} \times \left(\frac{a}{3}\right)^2}{6 \times \frac{\sqrt{3}}{4} \times (a)^2} = \frac{1}{3} \]

Q7. Question is incomplete.

Q8. A cube with edge of length 4 units is painted green on all the faces. The cube is then cut into 64 unit cubes. How many of these small cubes have

(i) 3 faces painted (ii) 2 faces painted (iii) one face painted (iv) no face painted

Ans: Number of cubes having

(i) 3 faces painted \( = 8 \) \{ At 8 corners \}
(ii) 2 faces painted \( = 24 \) \{ 12(n-2) \}
(iii) one face painted \( = 24 \) \{ 6(n-2)^2 \}
(iv) no face painted \( = 8 \) \{ (n-2)^3 \}

Q9. Let PQR be an equilateral triangle with each side of length 3 units. Let U,V, W,X,Y, and Z divide the sides into unit lengths. Find the ratio of the area UWXY (shaded) to the whole triangle PQR.
Ans: Length of altitude on PY from U = $\frac{\sqrt{3}}{2}$

Length of altitude on QW from U = $\sqrt{3}$

Shaded area = $ar(\Delta PQR) - ar(\Delta UPY) - ar(\Delta UQW) - ar(\Delta YXR)$

$$= \frac{\sqrt{3}}{4} \times 3^2 - \frac{1}{2} \times 2 \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times 1 \times \sqrt{3} - \frac{\sqrt{3}}{4} \times 1^2$$

$$= \sqrt{3} \text{ sq units}$$

Area of $\Delta PQR$ = $\frac{\sqrt{3}}{4} \times 3^2 = 9 \times \frac{\sqrt{3}}{4} \text{ sq units}$

∴ Fraction $= \frac{\frac{\sqrt{3}}{4} \times 1^2}{\frac{\sqrt{3}}{2} \times \sqrt{3}} = \frac{4}{9}$

Q10. Five houses P, Q, R, S and T are situated on the opposite side of a street from five other houses U, V, W, X and Y as shown in the diagram:

Houses on the same side of the street are 20 metres apart. A postman is trying to decide whether to deliver the letters using route PQRSTYXUWV or route PUQVRWSXTY, and finds that the total distance is the same in each case. Find the total distance in metres.

Ans: Let $PU=x$

Then $PQRSTYXUWV = (20+20+20+20+x+20+20+20) = 160+x$
And $x = \sqrt{400 + x^2} + x + \sqrt{400 + x^2} + x + \sqrt{400 + x^2} + x + \sqrt{400 + x^2} + x
\]

$= 5x + 4\sqrt{400 + x^2}$

Hence,

$5x + 4\sqrt{400 + x^2} = 160 + x$

$\Rightarrow 4\sqrt{400 + x^2} = 160 - 4x$

$\Rightarrow \sqrt{400 + x^2} = 40 - x$

$\Rightarrow 400 + x^2 = 1600 + x^2 - 80x$

$\Rightarrow 80x = 1200$

$\therefore x = 15$

$\therefore$ Total distance $= 175$ m

{ Alternate solutions and corrections if any are welcome }

**MATHEMATICS OLYMPIAD QUESTIONS**

1. Find the highest power of 3 which is contained in 100!

(Hint: If $p$ is prime number and $e$ is the largest experiment of $p$ such that $p^e / n!$, then $e = \sum \left\lfloor \frac{n}{p^i} \right\rfloor$).

(Ans: 48)

2. Find the highest power of 7 contained in 1000!

(Ans: 164)

3. Solve: $\log_2 x + \log_4 y + \log_4 z = 2$, $\log_3 y + \log_9 z + \log_9 x = 2$, $\log_4 z + \log_{16} x + \log_{16} y = 2$.

Ans: $x = 2/3$, $y = 27/8$, $z = 32/3$

4. Which is greater: $1.01^{1000000}$ or $100000$?

Ans: $1.01^{1000000} > 100000$

5. 6 X’s have to place in the squares of the figure given below such that each row containing at least one X. In how many different ways can this be done.
6. 28 games were played in a football tournament with each team playing once against each of the others. How many teams were there?

(Hint: \( n_{c_2} = 28 \))

Ans: \( n = 8 \)

7. If only downward motion along lines is followed, what is the total number of paths from point P to point Q in the figure given below.

Ans: 20

8. A printer numbers the pages of a book starting with 1. He uses 3189 digits in all. How many pages does the book have?

Ans: 1074

9. If \( x = 5 + 2\sqrt{6} \), find the value of \( \sqrt{x} + \frac{1}{\sqrt{x}} \)

Ans: 970

10. a) Solve: \( \frac{3}{2} (x - 1) + \sqrt{2x^2 - 7x - 4} \)

Ans: \( \frac{1}{\sqrt{2}} (\sqrt{2x + 1} + \sqrt{x - 4}) \)

b) If \( x + y + z = 3; x^2 + y^2 + z^2 = 5 \) and \( x^3 + y^3 + z^3 = 7 \), find the value of \( x^4 + y^4 + z^4 \).

Ans: 9
SAMPLE PAPERS
SUMMATIVE ASSESSMENT 1  
CLASS X

<table>
<thead>
<tr>
<th>Type of question</th>
<th>Marks per Question</th>
<th>Total No. of questions</th>
<th>Total Marks</th>
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<tr>
<td>SA-1</td>
<td>2</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>SA-II</td>
<td>3</td>
<td>10</td>
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</tr>
<tr>
<td>LA</td>
<td>4</td>
<td>11</td>
<td>44</td>
</tr>
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<td>TOTAL</td>
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<td>34</td>
<td>90</td>
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BLUE PRINT

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<thead>
<tr>
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<th>SA(1)</th>
<th>SA(II)</th>
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<td>3(1)</td>
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<td>12(3)</td>
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<tr>
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<td>2(1)</td>
<td>6(2)</td>
<td>8(2)</td>
<td>17(6)</td>
</tr>
<tr>
<td>TOTAL</td>
<td>4(4)</td>
<td>12(6)</td>
<td>30(10)</td>
<td>44(11)</td>
<td>90(31)</td>
</tr>
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</table>

Note: Number of question(s) are given within brackets and marks outside the brackets.
MATHEMATICS

SUMMATIVE ASSESSMENT – 1

CLASS: X

Max. Marks:90

TIME : 3 HRS.

General Instructions :
1. All questions are compulsory
2. The question paper consists of 31 questions divided into four sections A, B, C & D. Section-A contains 4 questions of 1 mark each, Section-B contains 6 questions of 2 marks each, Section-C contains 10 questions of 3 marks each and Section-D contains 11 questions of 4 marks each.
3. There is no overall choice and internal choice.
4. Use of calculator is not permitted.
5. An additional 15 minutes time has been allotted to read this question paper only.

SECTION - A

Question numbers 1 to 4 carry 1 mark each. 1 x 4 = 4

1. If \( x = 3 \sec^2 \theta \) and \( y = 3 \tan^2 \theta \), then find the value of \( x - y \).

2. In fig.1, DE || BC, if BD = 3 cm, AD = 2 cm, AE = 4 cm, then find the value of EC.

3. If one zero of the quadratic polynomial \( kx^2 + 3x + k \) is 2, then find the value of \( k \).

4. The median and mean of a frequency distribution are 24 and 28 respectively. Find the mode.

SECTION B

Question numbers 5 to 10 carry 2 marks each. 2 x 6 = 12

5. Given that HCF (306, 657) = 9, find LCM (306, 657).

6. If the lines given by \( 3x + 2Ky = 2 \) and \( 2x + 5y = -1 \) are parallel lines, then find the value of \( K \).

7. Use elimination method to find all possible solutions of the following pair of linear equations: \( 3x + 2y = 12 \) and \( 5x - 2y = 4 \).

8. After how many decimal places will the decimal expansions of the number \( \frac{53}{5^2 \times 2^3} \).

9. In \( \triangle ABC \), DE is a line such that D and E are points on AB and CA and \( \angle B = \angle AED \). Show that \( \triangle ABC \sim \triangle AED \).
10. Find the mode class and the median class for the following distribution:

<table>
<thead>
<tr>
<th>Class Interval</th>
<th>0 – 10</th>
<th>10 - 20</th>
<th>20 - 30</th>
<th>30 - 40</th>
<th>40 - 50</th>
</tr>
</thead>
<tbody>
<tr>
<td>frequency</td>
<td>6</td>
<td>10</td>
<td>12</td>
<td>8</td>
<td>7</td>
</tr>
</tbody>
</table>

**SECTION C**

Question numbers 11 to 20 carry 3 marks each.  
3 x 10 = 30

11. Find the mean of the given frequency distribution table:

<table>
<thead>
<tr>
<th>Class Interval</th>
<th>15-25</th>
<th>25-35</th>
<th>35-45</th>
<th>45-55</th>
<th>55-65</th>
<th>65-75</th>
<th>75-85</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>6</td>
<td>11</td>
<td>7</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

12. Prove that $5 + \sqrt{3}$ is irrational.

13. A taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 Km, the charge paid is Rs. 105 and for a journey of 15 Km, the charge paid is Rs. 155. What are the fixed charges and the charge per Km. How much does a person have to pay for travelling a distance of 25 Km.

14. Prove that $\frac{\cos \theta}{\csc \theta - 1} + \frac{\csc \theta}{\cos \theta + 1} = 2\sec^2 \theta$.

15. Prove that $\cos^2 90^\circ + \frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \cdot \csc 31^\circ = 2$.

16. In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

17. Find the zeros of the quadratic polynomial $x^2 - 2x - 8$ and verify the relationship between the zeros and the coefficients.

18. $\triangle ABC$ and $\triangle DBC$ are on the same base and on opposite sides of BC and O is the point of intersection of AD and BC. Prove that: $\frac{\text{area}(\triangle ABC)}{\text{area}(\triangle DBC)} = \frac{AO}{DO}$

19. Prove that $\frac{\tan \theta - \cot \theta}{\sin \theta \cdot \cos \theta} = \tan^2 \theta - \cot^2 \theta$
20. The percentage of marks obtained by 100 students in an examination are given below:

<table>
<thead>
<tr>
<th>Marks</th>
<th>30-35</th>
<th>35-40</th>
<th>40-45</th>
<th>45-50</th>
<th>50-55</th>
<th>55-60</th>
<th>60-65</th>
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<tbody>
<tr>
<td>frequency</td>
<td>14</td>
<td>16</td>
<td>18</td>
<td>23</td>
<td>18</td>
<td>8</td>
<td>3</td>
</tr>
</tbody>
</table>

Determine the mode percentage of marks.

SECTION D

**Question numbers 21 to 31 carry 4 marks each**

21. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

22. Find the other zeros of polynomial \( P(x) = 2x^3 - 3x^2 - 3x - 2 \) if two of its zeros are \( \sqrt{2} \) and \(-\sqrt{2}\).

23. If \( x = b \cos A - a \sin A \) and \( y = a \cos A + b \sin A \), then prove that \( x^2 + y^2 = a^2 + b^2 \).

24. Using Euclid’s Division Algorithm, show that the square of any positive integer is either of the form \( 3q \) or \( 3q + 1 \) for some integer \( q \).

25. Sides \( AB \) and \( BC \) and median \( AD \) of a triangle \( ABC \) are respectively proportional to sides \( PQ \) and \( QR \) and median \( PM \) of another triangle \( PQR \). Show that \( \Delta ABC \sim \Delta PQR \).

26. Show graphically \( x - y + 1 = 0 \) and \( 3x + 2y - 12 = 0 \) has a unique solution. Also, find the area of triangle formed by these lines with \( x \)-axis.

27. Prove that \( (\sin A + \csc A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A \)

28. The following distribution gives the annual profit earned by 30 shops of a shopping complex.

<table>
<thead>
<tr>
<th>Profit (in Lakh Rs.)</th>
<th>0 – 5</th>
<th>5 – 10</th>
<th>10 – 15</th>
<th>15 – 20</th>
<th>20 – 25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of shops</td>
<td>3</td>
<td>14</td>
<td>5</td>
<td>6</td>
<td>2</td>
</tr>
</tbody>
</table>

Change the above distribution to less than type distribution and draw its ogive.

29. Shalini and her friends decided to teach some children of slum areas near their house. The children were made to sit in rows. If one child in extra in a row, there would be 2 rows less. If one child is less in a row, there would be 3 rows more. Find the number of children educated.

30. Evaluate: \( \left( \frac{3 \cos 43^\circ}{\sin 47^\circ} \right)^2 - \frac{\cos 37^\circ \csc 53^\circ}{\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ} \)

31. The following table shows the data of the amount donated by 100 people in a blind school.
<table>
<thead>
<tr>
<th>Amount Donated (in Rs.)</th>
<th>Number of persons</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 100</td>
<td>2</td>
</tr>
<tr>
<td>100 - 200</td>
<td>5</td>
</tr>
<tr>
<td>200 - 300</td>
<td>X</td>
</tr>
<tr>
<td>300 - 400</td>
<td>12</td>
</tr>
<tr>
<td>400 - 500</td>
<td>17</td>
</tr>
<tr>
<td>500 - 600</td>
<td>20</td>
</tr>
<tr>
<td>600 - 700</td>
<td>Y</td>
</tr>
<tr>
<td>700 - 800</td>
<td>9</td>
</tr>
<tr>
<td>800 - 900</td>
<td>7</td>
</tr>
<tr>
<td>900 - 1000</td>
<td>4</td>
</tr>
</tbody>
</table>

If the median of the above data is 525, find the value of X and Y. What values are depicted here?
SUMMATIVE ASSESSMENT -II
CLASS X

MATHEMATICS

Time: 3 hours
M.M:90

General Instructions
1. All questions are compulsory.
2. The question paper consists of 31 questions divided into four sections A, B, C and D
3. Section A contains 4 questions of 1mark each, Section B contains 6 questions of 2 marks each,
Section C contains 10 questions of 3 marks each and Section D contains 11 questions of 4marks each.

SECTION A (1 MARK)

1. If the common difference of an AP is 3, then what is $a_{19} - a_{15}$?

2. Find the angle of elevation of the top of a tower $100\sqrt{3}$ m long, from a point at a distance of 100m, from
the foot of the tower in a horizontal plane.

3. Find the area of quadrant of a circle whose circumference is 22cm.

4. Find the value of k if the point P (2, 4) is equidistant from the points A(5, k) and B(k, 7)

SECTION B (2 MARKS)

5. Find the roots of the quadratic equation: $4\sqrt{3} x^2 + 5x - 2\sqrt{3} = 0$

6. Is (-217) a term of the A P: 27, 22, 17, 12, ……………….?

7. In fig. if length of AB is 9 cm, then, find the length of CP

8. A drinking glass is in the shape of a frustum of a cone of height 14cm. The diameters of its two circular
ends are 4cm and 2cm. Find the capacity of the glass.

9. An unbiased dice is thrown. Find the probability of getting
   (a) a multiple of 3
   (b) a prime number
10. Two coins are tossed simultaneously. Find the probability of getting at most one head.

**SECTION C (3 MARKS)**

11. Solve the following equation for $x$:
\[
\frac{16}{x} - 1 = \frac{15}{x + 1} \quad (x \neq 0, -1)
\]

12. The sum of the 5th and the 9th terms of the AP is 30. If its 25th term is three times its 8th term, find the AP.

13. Two circular pieces of maximum radii and equal area, touching each other are cut out from a rectangular cardboard of dimensions $14\,\text{cm} \times 7\,\text{cm}$. Find the area of the remaining cardboard.

14. A circle is inscribed in a $\square$ABCD where $\angle B = 90^0$. The circle touches the sides AB, BC, CD and DA at P, Q, R and S respectively. If AD = 24cm, AB = 30cm and DR = 8cm, find the radius of the circle.

15. The angle of elevation of an aero plane from a point on the ground is $60^0$. After a flight of 30 seconds, the angle of elevation changes to $30^0$. If the aero plane is flying at a constant height of $3000 \sqrt{3}\,\text{m}$, find the speed of the aero plane.

16. A wheel of diameter 42 cm, makes 240 revolutions per minute. Find:

(i) the total distance covered by the wheel in one minute.

(ii) the speed of the wheel in km/hr.

17. The surface area of a sphere of radius 5 cm is 5 times the curved surface area of a cone of radius 4 cm. Find the volume of the cone.

18. A sphere of diameter 6 cm is dropped into a cylindrical vessel partly filled with water. The diameter of the vessel is 12 cm. If the sphere is completely submerged, how much will the water level be raised?

19. Two vertices of a triangle ABC are A (-7, 6) and B (8, 5). If the midpoint of the side AC is at $\left(-\frac{5}{2}, 2\right)$, find the coordinate of vertex C. Also, find the area of $\triangle ABC$.

20. A point P divides the line segment joining the points A(3, -5) and B(-4, 8) such that $\frac{AP}{PB} = \frac{K}{1}$. If P lies on the line $x + y = 0$, then find the value of K.

**SECTION D (4 MARKS)**

21. A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

22. In a school, the students decided to plant trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section will plant, will be double of the class in which they are studying. There are 1 to 12 classes in the school and each class has 2 sections.

(i) How many trees were planted by students.

(ii) How this act will help ‘save the earth’ campaign?

(iii) What value is depicted by the students?

23. A circle is inscribed in an equilateral $\triangle ABC$ of side 12 cm. Find the radius of the inscribed circle and the area of shaded region. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)
24. Prove that the tangent at any point of a circle is perpendicular to the radius at the point of contact.

25. Construct a \( \triangle PQR \) in which \( PQ = 6\text{ cm} \), \( QR = 7\text{ cm} \), \( \angle Q = 60^\circ \). Construct a \( \triangle \) similar to the given \( \triangle \) whose sides are \( \frac{5}{4} \) of the corresponding sides of \( \triangle PQR \).

26. From the top of a 7m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Find the height of the tower.

27. A toy is in the form of a cone of radius 3.5cm mounted on a hemisphere of same radius. The total height of the toy is 15.5cm. Find the total surface area of the toy.

28. A card is drawn at random from a well-shuffled deck of playing cards. Find the probability that the card drawn is

   (i) a king or a jack
   (ii) a non-ace card
   (iii) a red card
   (iv) neither a king nor a queen.

29. The difference of two natural numbers is 5 and the difference of their reciprocals is \( \frac{1}{10} \). Find the numbers.

30. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of circle.

31. Show that the points A (2, -2), B (14, 10), C (11, 13) and D (-1, 1) are the vertices of a rectangle.
SUMMATIVE ASSESSMENT – II
SAMPLE PAPER -2

CLASS : X
MATHEMATICS
Max Marks:90

Time: 3hrs

General Instructions:

i) All the questions are compulsory
ii) The question paper consists of 31 questions divided into four sections –A,B,C and D. Section A contains 4 questions of 1 mark each, Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 11 questions of 4 marks each.
iii) Use of calculator is not permitted
iv) The question paper contains value based question
v) 15 minutes time has been allotted to read the question paper.

SECTION: A

1. Write the next term of the AP \(\sqrt{8}, \sqrt{18}, \sqrt{32} \ldots \ldots\)
2. TP and TQ are the two tangents to a circle with centre O such that \(\angle POQ = 110^\circ\). Find \(\angle PTQ\).

3. The height of a tower is 10m. What is the length of its shadow when Sun’s altitude is 30°?
4. A coin is flipped to decide which team starts the game. What is the probability of your team will start?

SECTION: B

5. Find the values of k for which the quadratic equation \(2x^2 + kx + 3 = 0\) has real equal roots.
6. A sector is cut from a circle of radius 10.5 cm. The angle of the sector is 60°. Find the perimeter of the sector. (use \(\pi = \frac{22}{7}\)).
7. Find the values of \(y\) for which the distance between the points P(2, –3) and Q(10, \(y\)) is 10 units.
8. Find the 20th term from the last term of the AP 3,8,13,.................,253 ?
9. A point P divides the join of A(5, –2) and B(9, 6) are in the ratio 3 : 1. Find the coordinates of P
10. From a point P 10 cm away from the centre of a circle, a tangent PT of length 8 cm is drawn. Find the radius of the circle.

SECTION: C

11. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.
12. Using quadratic formula find the value for ‘x’ in \(9x^2 – 6ax + (a^2 – b^2) = 0\).
13. Find the ratio in which the point P(m, - 5) divides the line segment joining the point A(–3, 5) and B(4, - 9). Also find the value of ‘m’.
14. An observer, 1.5 m tall, is 28.5 m away from a tower 30 m high. Find the angle of elevation of the top of the tower from his eye.

15. If AB, AC, PQ are tangents in the figure given below and AB = 5 cm, find the perimeter of triangle APQ.

![Diagram of a triangle with tangents](image)

16. It is given that AB = 5 cm, AC = 6 cm and BC = 7 cm. Construct a similar to ΔABC such that each of its side is \( \frac{2}{3} \) of the corresponding sides of ΔABC.

17. In Fig., a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. (Use \( \pi = 3.14 \))

![Diagram of a square inscribed in a quadrant](image)

18. The toy is shaped like a cone surmounted by a hemisphere. The entire toy is 15.5 cm in height and the radius of the top is 3.5 cm. Find the area of the toy (use \( \pi = \frac{22}{7} \))

19. A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread to form a platform 22 m by 14 m. Find the height of the platform (use \( \pi = \frac{22}{7} \)).

20. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting
   a) a red card.
   b) a face card.
   c) ‘2’ of spades.

**SECTION: D**

21. If A(−4, −2), B(−3, −5), C(3, −2) and D(2, 3) are the vertices of a quadrilateral, find the area of the quadrilateral ABCD.

22. Prove “The lengths of tangents drawn from an external point to a circle are equal”.

23. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 30 seconds, the angle of elevation changes to 30°. If the jet plane is flying at a constant height of \( 3600\sqrt{3} \) m, find the speed of the jet plane in meters/second.

24. In figure, l and m are two parallel tangents at A and B. The tangent at C makes an intercept DE between the tangent l and m. Prove that \( \angle DFE = 90^0 \).
25. The present rate of milk is ₹ 30 per litre, but Rahul supplies it to an orphanage at ₹ 22 per litre. The bucket containing milk is 21 cm in height with radii of its lower and upper ends as 8 cm and 20 cm respectively. How much does the orphanage pay to Rahul? What value is depicted by Rahul? (use \( \pi = \frac{22}{7} \))

26. The speed of a boat in still water is 8 km/hr. It can go 15 km upstream and 22 km downstream in 5 hours. Find the speed of the stream.

27. If the nth term of an AP is given by \( a_n = 9 - 5n \), write the AP. Also find the sum of the first 15 terms.

28. If (-5) is a root of the quadratic equation \( 2x^2 + p(x) - 15 = 0 \) and the quadratic equation \( p(x^2 + x) + k = 0 \) has equal roots, find the value of ‘p’ and ‘k’.

29. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of corresponding minor and major segments of the circle. (Use \( \pi = 3.14 \) and \( \sqrt{3} = 1.732 \))

30. A container shaped like a right circular cylinder having diameter 12 cm and height 160 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and radius 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

31. A bag contains 4 white, 6 red, 7 black, and 3 blue balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is
   (a) White
   (b) Not black
   (c) Neither white nor black
   (d) Either red or white
FORMATIVE ASSESSMENTS
FORMATIVE ASSESSMENT-1
SUB: MATHEMATICS
CLASS X
Time: 1\(\frac{1}{2}\) hours  
Max.Marks:40

1. Section A : each question carries 1 mark  
2. Section B : each question carries 2 marks  
3. Section C : each question carries 3 marks  
4. Section D : each question carries 4 marks

SECTION A
1. Find the prime factors of 176  
2. Find the number of zeros in the following graph y = p(x) intersect the x-axis.

\[ y = p(x) \]

SECTION B
3. Using Euclid’s Division Lemma, find the H.C.F of 135 and 225  
4. “The product of two consecutive positive integers is divisible by 2”. Is this statement true or false? Give reasons.
5. Find the zeros of the polynomial 100x^2 – 81

SECTION C
6. Find the zeros of the polynomial 4x^2– 3x – 1 by factorization method and verify the relations between the zeros and the coefficients of the polynomial  
7. Solve the pair of Linear Equations x + y = 5 and 2x – 3y = 5 by elimination method.
8. Prove that √3 is irrational.
9. Two numbers are in the ratio 5:6. If 8 is subtracted from each of the numbers, the ratio becomes 4:5. Find the numbers.
SECTION D

10. Show that any positive odd integer is of the form 6q+1, 6q+3, or 6q+5 where q is some integer.

11. What must be added to \( f(x) = 4x^4 + 2x^3 - 2x^2 + x - 1 \) so that the resulting polynomial is divisible by \( g(x) = x^2 + 2x - 3 \)?

12. Find all the zeros of \( 2x^4 - 3x^3 - 3x^2 + 6x - 2 \), if two of its zeros are \( \sqrt{2} \) and \( -\sqrt{2} \).

13. Solve graphically the pair of linear equations: \( 3x + y - 3 = 0 \), \( 2x - y + 8 = 0 \), write the coordinates of the vertices of the triangle formed by two lines with x-axis.

14. Solve: \( \frac{5}{x+1} - \frac{2}{y-1} = \frac{1}{2} \)

\[ \frac{10}{x+1} + \frac{2}{y-1} = \frac{5}{2}, \text{ x}
eq 1, y 
eq 1. \]

MARKING SCHEME
Class X
Mathematics
Formative Assessment 1

1. Finding Prime Numbers 7 and 11 1 mark
2. 3 zeros 1 mark
3. \( 225 = 135 \times 1 + 90 \)
   \( 135 = 90 \times 1 + 45 \)
   \( 90 = 45 \times 2 + 0 \) \( \frac{1}{2} \) mark
   HCF = (225,90)  45 \( \frac{1}{2} \) mark
4. Yes any two consecutive numbers will have one odd and one even number. Hence the product of two numbers will be divisible by 2 \( (1 + 1) \) mark
5. \( (10x + 9) \times (10x - 9) \) 1mark
   Zeros of polynomial area \( \frac{9}{10} \) and \( -\frac{9}{10} \) \( \frac{1}{2} + \frac{1}{2} \) mark
6. \( 4x^2 - 4x + x - 1 \)
   \( (4x + 1) \times (x - 1) \)
   Zeros are \( \frac{-1}{4} \) and 1 1mark
   Sum of the zeros = 1 with proper steps
   Product of the zeros = \( -\frac{1}{4} \) \( \frac{1}{2} + \frac{1}{2} \) mark
7. \( y = 5 - x \)
   Substituting and finding the value of \( y = 1 \) \( \frac{1}{2} + 1 \) mark
   Finding the value of \( x = 4 \) 1 mark
8. \( \sqrt{3} = \frac{p}{q} \) \( \frac{1}{2} \) mark
3 = \frac{p^2}{q^2}

\text{3 divides } p^2 \quad \frac{1}{2} \text{ mark}

3 divides p \quad \frac{1}{2} \text{ mark}

\text{Put } p = 3m \text{ and proving } 3 \text{ divides } q^2 \quad \frac{1}{2} \text{ mark}

\text{Proving } \sqrt{3} \text{ is irrational number with reason} \quad \frac{1}{2} \text{ mark}

9. Considering two numbers as 5x and 6x

\frac{5x-8}{6x-8} = \frac{4}{5} \quad 1 \text{ mark}

Finding x = 8 \quad \frac{1}{2} \text{ mark}

Writing 2 numbers as 40 and 48 \quad \frac{1}{2} \text{ mark}

10. Euclid division algorithm \( a = bq + r \) (0 ≤ r < b)

Putting r = 0, 1, 2, 3, 4, 5 finding 6q + 1 not divisible by 2 etc. \quad \frac{1}{2} \text{ mark}

Since 6q, 6q + 2, 6q + 4 are even

The remaining 6q + 1, 6q + 3, 6q + 5 are odd. \quad 1 \text{ mark}

11. Long division method and obtaining quotient as \( 4x^2 - 6x + 22 \)

\( R(x) = -61x + 65 \) \quad 1 \text{ mark}

61x – 65 must be added \quad 1 \text{ mark}

12. Divder of the polynomial \( (x + \sqrt{2}) (x - \sqrt{2}) = x^2 - 2 \)

Long division \quad 1 \frac{1}{2} \text{ mark}

Obtaining quotient as \( 2x^2 - 3x + 1 \) and factorization \quad 1

Writing other two zeros as 1 and \( \frac{1}{2} \) \quad \frac{1}{2} \text{ mark}

13. Writing \( x = -1, y = 6 \)

Coordinates of three vertices of triangle A(1,0), B(-4,0) and C(-1,6) \quad \frac{1}{2} \text{ mark}

Marking correctly in the graph \quad 1 \frac{1}{2} \text{ mark}

14. Considering \( \frac{1}{x+1} \) as a and \( \frac{1}{y-1} \) as b \quad 1 \text{ mark}

Substituting and solving and obtaining \( a = \frac{1}{5} \) and \( b = \frac{1}{4} \) \quad 2 \text{ mark}

Obtaining \( x = 4, y = 5 \) \quad 1 \text{ mark}
FORMATIVE ASSESSMENT – III
Class X - MATHEMATICS

Time: 1½ hours
Marks: 40

Answer all questions.

SECTION – A (1 X 2 = 2 MARKS)

1. Find the value of ‘k’ for which 3 is a root of the equation \( kx^2 - 7x + 3 = 0 \)
2. From a point Q, the length of the tangent to a circle is 24cm and the distance of Q from the centre is 25 cm. Find the radius of the circle.

SECTION – B (2 X 3 = 6 MARKS)

3. Find the value of ‘k’ for which the equation \( kx(x - 2) + 6 = 0 \) have two equal roots.
4. Draw a line segment of length 8cm and divide it in the ratio 2:3 internally.
5. Prove that the tangents drawn at the ends of diameter of a circle are parallel.

SECTION – C (3 X 4 = 12 MARKS)

6. Find the nature of roots of quadratic equation \( 2x^2 - 6x + 3 = 0 \) and if real roots exist, find them using quadratic formula.
7. Find the sum of first 51 terms of an AP whose 2\(^{nd}\) and 3\(^{rd}\) terms are 14 and 18 respectively.
8. Construct a triangle of sides 5cm, 6cm and 4cm and then another triangle whose sides are \( \frac{4}{3} \) of the corresponding sides of the first triangle.
9. \( \triangle ABC \) circumscribes a circle of radius 2cm such that BC is divided by the point of contact D. If BD = 4cm, DC = 3cm and the area of \( \triangle ABC \) is 21cm\(^2\), find the lengths of AB and AC.

SECTION – D (4 X 5 = 20 MARKS)

10. The tangent at any point of a circle is perpendicular to the radius through the point of contact-Prove.
11. A train travels 720km at a uniform speed. If the speed had been 8 km/hour more, it would have taken 3 hours less for the same journey. Find the speed of the train.
12. Draw a pair of tangents to a circle of radius 3 cm, which are inclined to each other at an angle of 60\(^{0}\).
13. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs.200 for the first day, Rs.250 for the second day, Rs.300 for the third day, etc., the penalty for each succeeding day being Rs. 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?
14. Which term of the AP: 3, 10, 17,….. will be 84 more than its 13\(^{th}\) term?
MARKING SCHEME

1. \( k = 2 \) ................. .......................... 1 mark

2. \( 7 \text{cm} \) ................. .......................... 1 mark

3. The equation is \( kx^2 - 2kx + 6 = 0 \) ................. 

For equal roots, \( D = 0 \)

\( b^2 - 4ac = 0 \) .......................... .......................... .......................... 1 mark

\( 4k^2 - 24k = 0 \) .......................... .......................... .......................... 1 mark

\( 4k - 24k^2 = 0 \)
\( 4k (k - 6) = 0 \)
\( k = 0 \), not possible, \( k - 6 = 0 \), or, \( k = 6 \) ........ .......................... 1 mark

4. Line segment and acute angle drawing........... .......

Making \( 2 + 3 = 5 \) equal arcs .......................... .......................... 1 mark

Completing the correct division and construction .......................... 1 mark

5. Figure .................................... .......................... .......................... 1 mark

Explaining/proving by alternate angles or co-interior angles........ .......................... 1 mark

6. \( D = b^2 - 4ac = (-6)^2 - (4 \times 2 \times 3) = 36 - 24 = 12 \) ................. .......................... 1 mark

\( D > 0 \), the equation has real distinct roots. ......................... .......................... 1 mark

\( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 + \sqrt{3}}{2}, \frac{3 - \sqrt{3}}{2} \) .......................... 1 mark

7. \( a_2 = 14, a_3 = 18, d = a_3 - a_2 = 4 \) ............... .......................... 1 mark

\( a_2 = a + d, \ a = 14 - 4 = 10 \) ............... .......................... 1 mark

\( S_n = \frac{n}{2} [2a + (n-1)d] \) .......................... .......................... 1 mark

\( = \frac{51}{2} [20 + (50 \times 4)] \) .......................... .......................... 1 mark

\( = 5610 \) .......................... .......................... 1 mark

8. First triangle .......................... .......................... 1 mark

Constructing similar triangle ...... .......................... 2 marks
9. Fig.

\[ \frac{1}{2} \text{ mark} \]

OD = OB = OC = radius perpendicular to tangents ………….

\[ \frac{1}{2} \text{ mark} \]

Tangents from an external point to a circle are equal.

\[ BD = BF = 4 \text{cm}, \quad CD = CE = 3 \text{cm}, \quad \text{Let AF} = \text{AE} = x \text{ cm} \] ………….

\[ \frac{1}{2} \text{ mark} \]

Area of \( \Delta ABC = \) Sum of areas of \( \Delta AOB, \Delta BOC \) and \( \Delta AOC = 21 \text{cm}^2 \)

\[ \frac{1}{2} \times (AB + BC + CA) = 21 \text{cm}^2 \]

\[ \frac{1}{2} \times 2 (4 + x + 7 + 3 + x) = 21 \text{cm}^2 \] ………….

\[ \frac{1}{2} \text{ mark} \]

14 + 2x = 21cm²

\[ X = 3.5 \text{cm} \] ………….

\[ \frac{1}{2} \text{ mark} \]

AB = 7.5cm and AC = 6.5cm ………….

\[ \frac{1}{2} \text{ mark} \]

10. Fig, Given, To prove, Const.

\[ \frac{1}{2} \text{ mark} \]

each

Proof ………….

2marks

11. Let the speed of train be \( x \) km/h………………

\[ \frac{1}{2} \text{ mark} \]
\[
\frac{720}{x} - \frac{720}{x+8} = 3 \quad \text{.................} \quad 1\text{mark}
\]

\[X^2 + 8x - 1920 = 0 \quad \text{...............} \quad 1\text{mark}\]

Solving and to get speed of train = 40 km/h \text{.................} \quad 1\frac{1}{2}\text{mark}

12. Drawing a circle and angle between 2 radii as 120° \text{.................} \quad 1\frac{1}{2}\text{mark}

Drawing perpendiculars at end points of radii \text{.................} \quad 1\frac{1}{2}\text{mark}

Meeting two tangents at external point \text{.................} \quad 1\text{mark}

13. A.P : 200, 250, 300, \text{.........................} \quad \frac{1}{2}\text{mark}

\[a = 200, \quad n = 30, \quad d = 50 \quad \text{...............} \quad 1\text{mark}\]

\[S_n = \frac{n}{2} [2a + (n-1)d] \quad \text{.........................} \quad \frac{1}{2}\text{mark}\]

\[= 15 [400 + 1450] \quad \text{.........................} \quad 1\text{.mark}\]

\[= 27750 \quad \text{.........................} \quad \frac{1}{2}\text{mark}\]

The contractor has to pay Rs. 27750 as penalty \text{.................} \quad \frac{1}{2}\text{mark}

14. \[a = 3, \quad d =7 \quad \text{...............} \quad \frac{1}{2}\text{mark}\]

\[a_n = a_{13} + 84 \quad \text{...............} \quad \frac{1}{2}\text{mark}\]

\[= a + 12d + 84 = 171 \quad \text{...............} \quad 1\text{mark}\]

\[a + (n - 1) d = 171 \quad \text{...............} \quad \frac{1}{2}\text{mark}\]

\[3 + (n - 1) d = 171 \quad \text{...............} \quad \frac{1}{2}\text{mark}\]

Solving to get \[n = 25 \quad \text{...............} \quad 1\text{mark}\]
Across

1. An instrument used to draw circles.
2. The shape of a circle.
4. The ratio of the circumference and diameter of any circle.
6. The number of square units occupied by the space inside the circle.
8. A part of a circle named by its endpoints.
10. A location in space that has no thickness.
11. The distance from the center of a circle to any point on the circle.
12. The distance around a circle.

Down

1. A line joining two points on the circle.
3. The distance across a circle through its center.
4. A circle divides a __________ into three parts.
5. Plural for half a diameter.
7. A circle has 360 of these units.
9. A shape with all points the same distance from its center.
13. All points in a circle are the same distance from this point.
MATHS QUIZ
1. Which is the fifth perfect number
2. In any right angled triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides. Who enunciated this theorem?
3. Name the shape of a mathematical figure a sugar crystal resemble
4. Name the theorem ‘’ The line joining the midpoints of two sides of a triangle is parallel to the third side and half its length’’
5. Name the angle subtended in a semicircle
6. Name the point of intersection of the three medians of a triangle
7. How many quadrants are there in a graph?
8. What is the sum of either pairs of opposite angles of a cyclic quadrilateral?
9. What is the probability of getting a prime number when a dice is thrown?
10. What will be the angle between the hands of a clock at the time 10.00 a.m?

COMPUTATIONAL SKILL
1. What is the square root of 0.09?
2. The sides of a right angle triangle are 5, 12, 13 cms. What is the area of the triangle?
3. If \( \sqrt{15} = 3.87 \) what is the value of \( \frac{\sqrt{15}}{3} \)
4. The area of a square is 4 sq cm. What is the length of its perimeter?
5. ABCD is a quadrilateral. The side BC is produced to E. If angle DCE = 108°. What is the measure of angle BAD?

PROBLEM SOLVING SKILL
1. The sum of two numbers is 50 and the difference is 10 what are those numbers?
2. Which is the next number in the pattern 2, 6, 12, 20, 30
3. It is a number. If we add 2/3 of the number to it, we get the sum as 35. What is that number?
4. What is the difference of \((a + b)^2\) and \((a - b)^2\)
5. The rate of 8mts cloth is Rs.20. Then what is the cost of 9mts cloth?
ANALYSING AND INTERPRETING SKILL

1. Six equal squares are kept side by side. Then a rectangle is formed. The perimeter of the rectangle is 308 cms. Then what is the area of each square in sq.cms?

2. A horse cart’s wheel has a radius of 84 cm. What is the circumference? If it moves 422.4mts how many revolution has it made?

3. Find the length of the largest rod that can be placed in a room measuring 12m x 9m x 8m

4. In the problem below, if + is x, - is ÷, ÷ is + and x is – what would be the answer of 16 ÷ 64 – 4 x 4 + 3

5. If the radius of a sphere is doubled, its volume will become how many times of its original volume?

Probability Worksheet

Rolling a Pair of Dice

A. Complete the table below.

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A. Complete the table below.
Introduction to Statistics

1. Find the range of each set of data.
   (a) 5, 10, 9, 7, 11, 8, 13
   (b) 1.3, 2.1, 1.8, 2.3, 2.2, 1.5
   (c) -14, +13, -15, 0, +12, -16, +11
   (d) 149, 218, 321, 0, 156, 223, 219

2. Find the median of each set of data.
   (a) 5, 10, 9, 7, 11, 8, 13
   (b) 1.3, 2.1, 1.8, 2.3, 2.2, 1.5
   (c) -14, +13, -15, 0, +12, -16, +11
   (d) 149, 218, 321, 0, 156, 223, 219
   (e) $215, $211, $246, $213, $287
   (f) 49, 62, 57, 44, 51, 67, 46

3. Find the mean of each set of data. Round decimal answers to the nearest tenth.
   a) 5, 10, 9, 7, 11, 8, 13
   b) 1.3, 2.1, 1.8, 2.3, 2.2, 1.5
   c) 149, 218, 321, 0, 156, 223, 219
   d) $215, $211, $246, $213, $287
   e) 49, 62, 57, 44, 51, 67, 46

4. Find the mode(s) of each set of data.
   (e) 5, 10, 9, 7, 10, 8, 13
   (f) 1.3, 2.1, 1.8, 2.3, 2.2, 2.1
   (g) -14, +13, +15, 0, +12, -14, +15
   (a) 149, 218, 321, 0, 156, 223, 219

LINK FOR ONLINE LABS (OLABS)
http://www.olabs.co.in/

POWER POINT ON EDMODO - THE TEACHER STUDENT ONLINE PLATFORM
TIPS AND TECHNIQUES IN TEACHING LEARNING PROCESS

1. Teach a Limited Number of *Math Facts* at a Time

   There is a big difference between figuring out the answer and memorizing facts. If students have too many facts to learn at one time, they necessarily fall back on figuring out the answer. Instead, we want students to REMEMBER the answers without having to figure them out.

2. Only add more facts to learn as the previous set has been mastered

   Once students have learned a set of facts to mastery, it is now possible to add two or three more facts to be learned. Student success is greatest when they have only two or three things to learn in a sea of material they have already mastered.

3. Practice should be cumulative

   Practice must be structured in a way that facts which have been previously deemed mastered continue to appear along with the two or three new facts that are being learned.

4. Students should memorize facts in a way that forms a verbal chain

   Students should always practice by saying the whole problem and the answer aloud. In this way, students memorize a verbal chain. As a result of this kind of practice, students hear/see 8x7 and can’t stop from saying/writing 56.

5. Mastery = automaticity

   Automaticity is the ability to say the answer to a problem immediately after reading the fact. There should be no hesitation.

6. Students should have realistic, individual fluency goals

   Students write at different speeds. The speed at which students can write the answer to facts is limited by their writing skill. Automaticity means that students should write answers to math facts exactly as fast they can write.

7. A routine for daily practice sessions should be in place

   Practice is most effective when it is distributed throughout the year. Learning math facts is just one part of a math program therefore efficient use of limited time is critical. A standard daily routine is necessary in order to accomplish this goal.

8. A routine for corrective feedback during practice should be in place

   Whenever students hesitate or give an incorrect answer, a corrective procedure should give them the answer, ensure they know it and provide them with a delayed test. It is very important that the delayed test occur before the student has a chance to forget the correct response.
9. Practice sessions should be short

Students cannot maintain focus on drill for more than 2-4 minutes at a time. Practice sessions may occur more than one time during the day, but should remain short.

10. A process for progress monitoring should be in place

If students are really learning math facts, the number of facts they can answer within a set time period should gradually increase. Periodically, students should be given a timed test of all the facts in the operations they are learning to see if their fluency is improving.

11. If students are to keep up with their grade level math program, they must begin memorizing multiplication facts in Grade 4 at the latest

Because fractions demand instantaneous recognition of multiplication facts, these must be mastered before fractions can be successfully learned. Even those students who need their fingers to add and subtract in Grade 4 and above, should learn multiplication facts to mastery or their continued progress in math is in jeopardy.

12. Implementing collaborative learning strategies as part of their everyday curriculum.

Collaborative learning is a process where students work together using each other’s strengths. Students get the opportunity to actively engage with their classmates, and learning flourishes through conversations. Students are challenged both socially and academically as they work together, listen to their peers’ perspectives, and articulate their thoughts. Creating an effective collaborative learning environment takes a lot of planning. Here are a few tips on how to make it work in your classroom.

13. Make Sure All Students Are Accountable for their Work

Effective collaborative learning groups are successful when everyone in the group is accountable for their work. Oftentimes students that are born leaders tend to overpower the group with their knowledge and strengths, leaving the others feeling like they can’t contribute. Or, you may have a situation where a student is lazy and doesn’t want to do the work, and then the other students feel like they have to carry all the weight of the group. Whatever the case may be, be sure to create an environment where all students are accountable, not just a few. Try cooperative learning strategies where each student has a specific role for their group, or where groups work in pairs within the group.

14. Create the Right Balance of Groups

Really take the time and think about how you want to put your groups together. In order for students to work well together, there needs to be the right balance. You can choose to group students by varying skill levels, gender, personality, you name it. Just be aware of what works and what doesn’t work. You can change groups as much or as little as you want to. Just make sure that you check in with students and see how they feel about the groups too.

15. Distribute Discipline Guidelines

The key to successful collaborative groups is all about the discipline. If you have rules set in place, then the odds of you having a positive classroom atmosphere are much greater. Before you even put students into groups explain your behavior expectations and consequences. Closely monitor each
group to see if they are working well together. Stop any disruptions or inappropriate behavior as soon as you spot it and be sure to stick to your consequences. When you enforce the rules and procedures then you will find students will behave and enjoy the collaborative learning environment.

Collaborative learning takes a lot of time and effort. Don’t get discouraged if it doesn’t seem like it’s working right away. Give it some time, let students adjust, and look and see what is working and what isn’t working. Once you get the groups situated, you will find that the students will really enjoy them.

As teachers, we all know that building your vocabulary can have many benefits. Children who are given the opportunity to learn new words not only improve their vocabulary and communication skills, but have an increased academic success rate.

Acquiring a large vocabulary will enable people to understand you better as well as help you get your point across much easier. With students, we can teach them plethora of vocabulary and have that be our main focus. But, it’s also important to incorporate “words” in everything that you teach.

16. Introduce each day a new Mathematical vocabulary

Have a word of the day. This is a great way to expand students’ Mathematics vocabulary. As soon as students enter the classroom, have a word of the day already on the front board. The students’ job is look the word up and write down the definition. Each day they come into school, they learn a new word and add it to their journals. At the end of the week, randomly choose one word that they learned throughout the week and have students tell you what it means as well as use it in a sentence. This will not only help to expand their vocabulary, but it will also keep them on their toes!

    By expanding students’ mathematics vocabulary, you will strengthen their use of the Mathematics language as well as increase the likelihood that they will do better in school. Celebrate words and make them a part of each and every day. Encourage students to have a love for word

17. Motivating the students

Motivating students to be (enthusiastically) receptive is one of the most important aspects of mathematics instruction and a critical aspect of the Common Core State Standards. Effective teachers should focus attention on the less interested students as well as the motivated ones. Presented in this blog post are nine techniques, based on intrinsic and extrinsic motivation, which can be used to motivate secondary school students in mathematics

**Extrinsic and Intrinsic Motivation**

**Extrinsic motivation** involves rewards that occur outside the learner's control. These may include token economic rewards for good performance, peer acceptance of good performance, avoidance of "punishment" by performing well, praise for good work and so on.

However, many students demonstrate **intrinsic goals** in their desire to understand a topic or concept (task-related), to outperform others (ego-related), or to impress others (social-related). The last goal straddles the fence between intrinsic and extrinsic.

With these basic concepts in mind, there are specific techniques which might be expanded, embellished and adapted to the teacher's personality and, above all, made appropriate for the learner's level of ability and environment. The strategies are the important parts to remember -- examples are provided merely to help understand the techniques.
Strategies for Increasing Student Motivation in Math

1. Call Attention to a Void in Students' Knowledge
This motivational technique involves making students aware of a void in their knowledge and capitalizes on their desire to learn more. For instance, you may present a few simple exercises involving familiar situations, followed by exercises involving unfamiliar situations on the same topic. The more dramatically you do this, the more effective the motivation.

2. Show a Sequential Achievement
Closely related to the preceding technique is that of having students appreciate a logical sequence of concepts. This differs from the previous method in that it depends on students' desire to increase, but not complete, their knowledge. One example of a sequential process is how special quadrilaterals lead from one to another, from the point of view of their properties.

3. Discovering a Pattern
Setting up a contrived situation that leads students to "discovering" a pattern can often be quite motivating, as they take pleasure in finding and then "owning" an idea. An example could be adding the numbers from 1 to 100. Rather than adding in sequence, students add the first and last (1 + 100 = 101), and then the second and next-to-last (2 + 99 = 101), and so on. Then all one has to do to get the required sum is multiplying 50 X 101 = 5,050. The exercise will give students an enlightening experience.

4. Present a Challenge
When students are challenged intellectually, they react with enthusiasm. Great care must be taken in selecting the challenge. The problem (if that is the type of challenge) must definitely lead into the lesson and be within reach of the students' abilities.

5. Entice the Class with a “Gee-Whiz” Mathematical Result
To motivate basic belief in probability, a very effective motivation is a class discussion of the famous "Birthday Problem," which gives the unexpectedly high probability of birthday matches in relatively small groups. It’s amazing -- even unbelievable -- result will leave the class in awe.

6. Indicate the Usefulness of a Topic
Introduce a practical application of genuine interest to the class at the beginning of a lesson. For example, in the high school geometry course, a student could be asked to find the diameter of a plate where all the information he or she has is a section smaller that a semicircle. The applications chosen should be brief and uncomplicated to motivate the lesson rather than detract from it.

7. Use Recreational Mathematics
Recreational motivation consists of puzzles, games, paradoxes or facilities. In addition to being selected for their specific motivational gain, these devices must be brief and simple. An effective execution of this technique will allow students to complete the "recreation" without much effort.

8. Tell a Pertinent Story
A story of a historical event (for example, math involved in building the Brooklyn Bridge) or contrived situation can motivate students. Teachers should not rush while telling the story. A hurried presentation minimizes the potential motivation of the strategy.
9. Get Students Actively Involved in Justifying Mathematical Curiosities

One of the more effective techniques for motivating students is asking them to justify one of many pertinent mathematical curiosities. The students should be familiar and comfortable with the mathematical curiosity before you "challenge" them to defend it.

Teachers of mathematics must understand the basic motives already present in their learners. The teacher can then play on these motivations to maximize engagement and enhance the effectiveness of the teaching process. Exploiting student motivations and affinities can lead to the development of artificial mathematical problems and situations. But if such methods generate genuine interest in a topic, the techniques are eminently fair and desirable.

TIPS TO MAKE CALCULATIONS EASY.

- **The 11 Times Trick**
  - We all know the trick when multiplying by ten – add 0 to the end of the number, but did you know there is an equally easy trick for multiplying a two digit number by 11? This is it:
  - Take the original number and imagine a space between the two digits (in this example we will use 52:
    - 5_2
  - Now add the two numbers together and put them in the middle:
    - 5_(5+2)_2
  - That is it – you have the answer: 572.
  - If the numbers in the middle add up to a 2 digit number, just insert the second number and add 1 to the first:
    - 9_(9+9)_9
    - (9+1)_8_9
    - 10_8_9
    - 1089

- **Quick Square**
  - If you need to square a 2 digit number ending in 5, you can do so very easily with this trick. Multiply the first digit by itself + 1, and put 25 on the end. That is all!
    - $25^2 = (2 \times (2+1)) \& 25$
    - $2 \times 3 = 6$
    - 625

- **Multiply by 5**
  - Most people memorize the 5 times tables very easily, but when you get in to larger numbers it gets more complex – or does it? This trick is super easy.
    - Take any number, then divide it by 2 (in other words, halve the number). If the result is whole, add a 0 at the end. If it is not, ignore the remainder and add a 5 at the end. It works everytime:
      - $2682 \times 5 = (2682 / 2) \& 5$ or 0
      - $2682 / 2 = 1341$ (whole number so add 0)
      - 13410
Let’s try another:

- \(5887 \times 5\)
- \(2943.5\) (fractional number (ignore remainder, add 5)

29435 is the answer

- **Multiply by 9**
  - This one is simple – to multiple any number between 1 and 9 by 9 hold both hands in front of your face – drop the finger that corresponds to the number you are multiplying (for example 9\(\times 3\) – drop your third finger) – count the fingers before the dropped finger (in the case of 9\(\times 3\) it is 2) then count the numbers after (in this case 7) – the answer is 2

- **Dividing by 5**
  - Dividing a large number by five is actually very simple. All you do is multiply by 2 and move the decimal point:
    - \(195 / 5\)
      - Step1: 195 * 2 = 390
      - Step2: Move the decimal: 39.0 or just 39
    - \(2978 / 5\)
      - step 1: 2978 * 2 = 5956
      - Step2: 595.6

- **Subtracting from 1,000**
  - To subtract a large number from 1,000 you can use this basic rule: subtract all but the last number from 9, then subtract the last number from 10:
    - 1000
      - -648
    - step1: subtract 6 from 9 = 3
    - step2: subtract 4 from 9 = 5
    - step3: subtract 8 from 10 = 2
  - answer: 352
TIPS FOR IMPROVING THE PERFORMANCE OF STUDENTS IN CLASS 10

GENERAL TIPS

1. Remedial classes after first unit test in class X must be started so as to get zero supplementary cases in Math and also to have a strong base in concepts.
2. Remedial class from the month of April onwards.
3. After the supplementary examination the lessons taught during their absence/preparation time must be taught again.
4. From the beginning of the session above average students must be trained in HOTS at least once in a week.
5. From the beginning of the session average students must be trained in Text book questions at least twice in a week.
6. Slips test based on one or two concepts with variation in questions can be given periodically.
7. During remedial classes chapter wise level 1 questions must be worked out by the students.
8. Two sets of question papers can be given for every formal test to avoid copying.
9. Answers of formal test must be discussed and the same may be worked out in the H.W note book.
10. Test papers must be always in the board pattern.
11. Basic concepts must be recalled before the beginning of Lesson without discouraging the students.

12. Revision must be given before every test, stressing on the Board questions.
13. Sure shot questions in 4 marks must be drilled.

14. During revision time after the first revision exam the students can be divided into three groups below average, average and above average. A time table can be framed including the other subjects with principal’s permission to focus on the three categories of the students.

15. Chapter wise formulae maybe prepared for a quick revision

16. Minimum five board question papers must be worked out.

17. From each chapter 1 mark board questions can be drilled.

18. The students must be insisted to write the values in the value based questions.

18. Never discourage the students.
ANNEXURES

ANNEXURES-Embedded Files:

LIST OF ANNEXURES CHAPTERWISE:

ANNEXURE 1: REAL NUMBERS

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ANNEXURE 2: POLYNOMIALS

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ANNEXURE 3: LINEAR EQUATION IN TWO VARIABLES

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ANNEXURE 4: QUADRATIC EQUATIONS

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ANNEXURE 5: ARITHMETIC PROGRESSION

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ANNEXURE 6: SIMILAR TRIANGLES

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### ANNEXURE 8: INTRODUCTION TO TRIGNOMETRY

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### ANNEXURE 9: APPLICATIONS OF TRIGNOMETRY

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</tbody>
</table>

### ANNEXURE 14: STATISTICS

<table>
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<tr>
<th>Sl. No</th>
<th>Software</th>
<th>Linked/embedded Documents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>VUE</td>
<td>1.STATISTICS .VUE</td>
</tr>
<tr>
<td>2.</td>
<td>PPT</td>
<td>1. STATISTICS.PPTX</td>
</tr>
</tbody>
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### ANNEXURE 15: PROBABILITY

<table>
<thead>
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<th>SOFTWARE</th>
<th>Linked/embedded Documents</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>VUE</td>
<td>1 priya-probability.vue</td>
</tr>
<tr>
<td>2</td>
<td>PPT</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>2.PPT2</td>
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</tbody>
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